Improved delay-range-dependent stability criteria for linear systems with interval time-varying delays

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Abstract: This paper provides new delay-range-dependent stability criteria in the forms of LMIs for systems with interval time-varying delays. Two cases concerning whether the derivative of the time delay is known are studied. A new estimation method is developed to estimate the nonlinear time-varying coefficients derived from Jenson's integral inequality more tightly than the existing ones. Along with convex combination and delay partitioning method, less conservative stability criteria are provided by utilizing the new estimation method. Numerical examples are given to illustrate the effectiveness of the proposed method.

Keywords: Time-varying delay, Lyapunov-Krasovskii functional, Delay-range-dependent stability, Linear matrix inequality (LMI)

1. Introduction

Time delay is encountered in many dynamic systems such as networked control systems, neutral networks and process control systems. In practical industrial systems, factors like the transmission of signals may lead to time delay. During the past few decades, much effort has been paid to the stability analysis of time-delay systems. It is generally accepted that delay-dependent stability criterion is less conservative than delay-independent ones especially when the size of the delay is small; so much attention has been paid to the study of delay-dependent stability criteria, see [1,2] for example.

There are four commonly used techniques to obtain delay-dependent stability criteria. They include the model transformation, the bounding inequalities of the cross terms and integral terms, the free-weighting matrices method, and the convex combination method. The basic principle of the model transformation method is to utilize the Newton-Leibniz formula to transform discrete delay of the system into distributed delay. Firdman et al. summarized four model transformation methods of linear time-invariant delay system in [1]. The bounding techniques of the cross terms and integral terms in the derivatives of the Lyapunov-Krasovskii functional (LKF) are widely investigated, including Park inequality [3], Moon inequality [4] and Jeson's inequality [5]. Some other inequalities (C. Peng et al. [6]) have been employed to deal with the triple integral terms in the derivative of the LKF by Sun in [7, 8].The free-weighting matrices method is proposed by He et al. [9]. By utilizing the Newton-Leibniz formula, free-weighting matrices are introduced into the derivative of the Lyapunov functional to reduce the conservatism led by model transformation and matrix inequalities. Combining the free weighting matrices method with different kinds of LKF, better stability criteria is derived, see for examples in [10-11]. However, this method increases the computational complexity since many slack variables are added into the LMIs. The convex combination method has been introduced in [12, 13] to obtain some less conservative stability results.

In [12], Shao estimated the term $-(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}^T(s)Z\dot{x}(s)ds$ by using a form of convex combination instead of simply enlarge it as $-(h_2-h_1)\int_{t-\tau(t)}^{t-h_1}\dot{x}^T(s)Z\dot{x}(s)ds$, where h_1 and h_2 are the lower and upper bounds of the time-varying delay $\tau(t)$ respectively, and $x(t)$ is the state vector of the system. To deal with the nonlinear time-varying coefficients $\frac{h_2-h_1}{\tau(t)-h_1}$ and $\frac{h_2-h_1}{h_2-\tau(t)}$ derived from the Jenson's inequality, a tighter estimation based on the delay partitioning idea and the convex combination is proposed by Zhu et al. [13], in which the inequality $\frac{h_2-h_1}{\tau(t)-h_1}+\frac{h_2-h_1}{h_2-\tau(t)} \geq \frac{2N}{i+1}+\frac{2N}{2N-(i+1)}, i=0,1,\ldots,N-1$ with respect to $\tau(t)\in[h_1+\frac{h_2-h_1}{2N}\times i, h_1+\frac{h_2-h_1}{2N}\times(i+1)]$ is used.

Note that in [13] the estimation of $\frac{h_2-h_1}{\tau(t)-h_1}+\frac{h_2-h_1}{h_2-\tau(t)}$ is obtained by some constants, which leaves some room for improvement. In this paper, by introducing an additional time-varying term to the estimation of $\frac{h_2-h_1}{\tau(t)-h_1}+\frac{h_2-h_1}{h_2-\tau(t)}$, a tighter bound of the nonlinear time-varying coefficients can be obtained so that less conservative stability criteria are derived for two cases concerning whether the derivative of the time delay is known or not. The stability criteria are in the forms of LMIs, by solving which the admissible bounds of the time-varying delay can be obtained. Numerical examples are given to demonstrate the effectiveness of the proposed method.

The organization of the remaining part is as follows. In Section 2, some lemmas used in this paper are presented. In Section 3, the main result is established. In Section4, numerical simulation examples are given for illustration. Finally, conclusions are stated in Section 5.

2. Problem formulation and preliminaries

Consider the following linear system with time-varying delay,

$$\begin{cases} \dot{x}(t)=Ax(t)+A_1x(t-\tau(t)), t\geq 0 \\ x(t)=\phi(t), t\in[-h_2,0] \end{cases} \tag{1}$$

where $x(t)\in\Re^n$ is the state vector; the initial condition $\phi(t)$ is a continuously differentiable vector-valued function; $A\in\Re^{n\times n}$ and $A_1\in\Re^{n\times n}$ are constant system matrices; $\tau(t)$ is a time-varying differentiable function which satisfies

$$0\leq h_1\leq\tau(t)\leq h_2, \tag{2}$$

and

$$\dot{\tau}(t)\leq\mu, \tag{3}$$

where $0\leq h_1<h_2$, and $0\leq\mu$ are constants.

Let $h_{12} = h_2 - h_1$. The following lemmas are introduced which are important in the derivation of the main result.

Lemma 1. (Jenson's Inequality [5]) Suppose $0 \le h_1 \le \tau(t) \le h_2$ and $x(t) \in \Re^n$, for any positive matrix $R \in \Re^{n\times n}$, the following inequality holds,

$$-(h_2 - h_1)\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R\dot{x}(s)ds \le \begin{bmatrix} x(t-h_1) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ x(t-h_2) \end{bmatrix}.$$

Lemma 2. Suppose $0 < A < B \le \frac{1}{2}$ are constants and $\alpha(t) \in [A, B]$, then

$$\frac{1}{\alpha(t)} + \frac{1}{1-\alpha(t)} \ge \frac{1}{1-\alpha(t)}[1 - \frac{(1-B)^2}{B^2}] + \frac{1}{B^2}. \tag{4}$$

Proof: Denote $f_1(\alpha(t)) = \frac{1}{\alpha(t)} + \frac{1}{1-\alpha(t)}$ and $f_2(\alpha(t)) = \frac{1}{1-\alpha(t)}[1 - \frac{(1-B)^2}{B^2}] + \frac{1}{B^2}$.

It is obvious that when $\alpha(t) = B$, $f_1(\alpha(t)) = f_2(\alpha(t)) = \frac{1}{B^2}$ holds.

Taking the derivative of $f_1(\alpha(t))$ and $f_2(\alpha(t))$ along α, we have

$\dot{f}_1(\alpha) = -\frac{1}{\alpha^2} + \frac{1}{(1-\alpha)^2}$ and $\dot{f}_2(\alpha) = -\frac{1}{(1-\alpha)^2}\left[\frac{(1-B)^2}{B^2} - 1\right]$, thus,

$$\dot{f}_1(\alpha) - \dot{f}_2(\alpha) = -\frac{1}{\alpha^2} + \frac{1}{(1-\alpha)^2} \times \frac{(1-B)^2}{B^2}.$$

With respect to $\alpha(t) \in [A, B]$, we have $\dot{f}_1(\alpha) \le \dot{f}_2(\alpha)$, and $f_1(\alpha(t)) = \frac{1}{\alpha(t)} + \frac{1}{1-\alpha(t)}$ is strictly decreasing because $0 < \alpha \le \frac{1}{2}$. So we have (4). This completes the proof. ■

Remark 1. As illustrated in **[13]**, the estimation $\frac{1}{\alpha(t)}+\frac{1}{1-\alpha(t)}\geq 3$ is used in [8] and [12]. In **[13]**, it is estimated as constants $\frac{1}{B_i}+\frac{1}{1-B_i}$ in different subintervals, while in Lemma 2, the minimum value of $\frac{1}{1-\alpha(t)}[1-\frac{(1-B)^2}{B^2}]+\frac{1}{B^2}$ is $\frac{1}{B_i}+\frac{1}{1-B_i}$. The estimation (4) is tighter than the existing ones. This is also illustrated in Fig. 1.

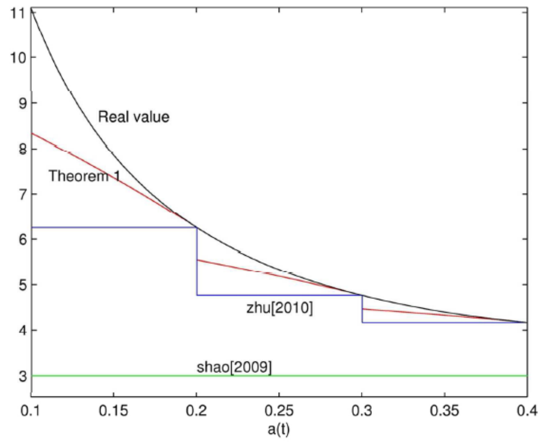

Fig. 1. Different method of estimation of $\frac{1}{\alpha(t)}+\frac{1}{1-\alpha(t)}$

From Fig. 1, it is obvious that the estimation method developed in this paper gets a tighter bound of $\frac{1}{\alpha(t)}+\frac{1}{1-\alpha(t)}$. When the numbers of subintervals increases, the estimation approaches the true value.

3. Main results

Now it is ready to investigate the stability problem of system (1).

Theorem 1. Given scalars $0 \le h_1 < h_2$, $0 \le \mu$, and positive integer $N \ge 1$, the system (1) with a time varying delay is asymptotically stable if there exist matrices $P > 0$, $Q_i > 0$, $i = 1,2,3$, $Z_j > 0$, $j = 1,2$ with appropriate dimensions such that the following LMIs hold.

$$\phi_i^1 = \phi - \gamma_i^1 \psi_1 - \gamma_i^2 \psi_2 < 0, \tag{5}$$

and

$$\phi_i^2 = \phi - \gamma_i^2 \psi_1 - \gamma_i^1 \psi_2 < 0, \tag{6}$$

where $i = 0,1,2..., N-1$, and

$$\gamma_i^1 = \left(\frac{2N}{i+1}\right)^2 - \frac{2N}{2N-i} \times \left(\frac{2N-i-1}{i+1}\right)^2,$$

$$\gamma_i^2 = \frac{2N}{2N-i},$$

$$\phi = \begin{bmatrix} PA + A^T P + Q_1 + Q_2 + Q_3 - Z_1 & PA_1 & Z_1 & 0 \\ * & -(1-\mu)Q_3 & 0 & 0 \\ * & * & -Q_1 - Z_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix}$$

$$+ [A \quad A_1 \quad 0 \quad 0]^T (h_1^2 Z_1 + h_{12}^2 Z_2)[A \quad A_1 \quad 0 \quad 0],$$

$$\psi_1 = [0 \quad -I \quad I \quad 0]^T Z_2 [0 \quad -I \quad I \quad 0],$$

$$\psi_2 = [0 \quad -I \quad 0 \quad I]^T Z_2 [0 \quad -I \quad 0 \quad I].$$

Proof. Construct a Lyapunov functional as

$$V(t, x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t), \tag{7}$$

where

$$V_1(x_t) = x^T(t)Px(t)\,, \tag{8}$$

$$V_2(x_t) = \int_{t-h_1}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_2}^{t} x^T(s)Q_2x(s)ds + \int_{t-\tau(t)}^{t} x^T(s)Q_3x(s)ds\,, \tag{9}$$

$$V_3(x_t) = h_1\int_{-h_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\theta + h_{12}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\theta\,. \tag{10}$$

Denote $\xi(t) = col\{x(t), x(t-\tau(t)), x(t-h_1), x(t-h_2)\}$.

Taking the time derivative of $V(t, x_t)$ along the trajectory of system (1) yields

$$\dot{V}(t, x_t) = \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t)\,,$$

$$\dot{V}_1(x_t) = 2x^T(t)P\dot{x}(t) = 2x^T(t)P[Ax(t) + A_1x(t-\tau(t))]\,, \tag{11}$$

$$\dot{V}_2(x_t) \le x^T(t)(Q_1 + Q_2 + Q_3)x(t) - x^T(t-h_1)Q_1x(t-h_1))$$

$$-x^T(t-\tau(t))(1-\mu)Q_3x(t-\tau(t)) - x^T(t-h_2)Q_2x(t-h_2)\,, \tag{12}$$

$$\dot{V}_3(x_t) = \dot{x}^T(t)(h_1^2Z_1 + h_{12}^2Z_2)\dot{x}(t) - h_1\int_{t-h_1}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - h_{12}\int_{t-h_2}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)ds$$

$$\le \dot{x}^T(t)(h_1^2Z_1 + h_{12}^2Z_2)\dot{x}(t) - \xi^T(t)(e_1 - e_3)Z_1(e_1^T - e_3^T)\xi(t)$$

$$-h_{12}\int_{t-h_2}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)ds\,. \tag{13}$$

From (11) ~ (13), we can have

$$\dot{V}(t, x_t) \le \xi^T(t)\phi\xi(t) - h_{12}\int_{t-h_2}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)ds\,.$$

Using Lemma 1, we can obtain

$$\dot{V}(t, x_t) \le \xi^T(t)(\phi - \frac{h_2 - h_1}{\tau(t) - h_1}\psi_1 - \frac{h_2 - h_1}{h_2 - \tau(t)}\psi_2)\xi(t)\,. \tag{14}$$

Denoting $\alpha(t)=\frac{\tau(t)-h_1}{h_2-h_1}$, we divide the delay interval $[h_1,h_2]$ into 2N segments of the same length $\delta=\frac{h_2-h_1}{2N}$, where *N* is a positive integer. Now we will estimate the integral terms derived by the Jenson's inequality in *2N* intervals for the following two cases.

Case 1: For the case that $h_1+i\times\delta\le\tau(t)\le h_1+(i+1)\times\delta$, $i=0,1,2...,N-1$, we have $\alpha(t)\in[\frac{i}{2N},\frac{i+1}{2N}]$. Denote $A_i=\frac{i}{2N}$, $B_i=\frac{i+1}{2N}$.

From Lemma 2, it follows that

$$\frac{1}{\alpha(t)}\ge\frac{1}{1-\alpha(t)}\left[1-\frac{(1-B_i)^2}{B_i^2}\right]+\frac{1}{B_i^2}-\frac{1}{1-\alpha(t)}=-\frac{1}{1-\alpha(t)}\times\frac{(1-B_i)^2}{B_i^2}+\frac{1}{B_i^2}. \tag{15}$$

Note that $\left(-\frac{1}{1-\alpha(t)}\times\frac{(1-B_i)^2}{B_i^2}+\frac{1}{B_i^2}\right)\psi_1+\frac{1}{1-\alpha(t)}\psi_2$ is a convex combination of ψ_1 and ψ_2. Therefore substituting $\alpha(t)=A_i$ and $\alpha(t)=B_i$, we can get

$$\phi_i^1=\phi-\gamma_i^1\psi_1-\gamma_i^2\psi_2<0, \tag{16}$$

and

$$\phi-\frac{1}{B_i}\psi_1-\frac{1}{1-B_i}\psi_2<0. \tag{17}$$

So we have that if (16) and (17) holds, then $\phi-\frac{1}{\alpha(t)}\psi_1-\frac{1}{1-\alpha(t)}\psi_2<0$, $\forall\alpha(t)\in[A_i,B_i]$.

Case 2: For the case $h_2-(i+1)\times\delta\le\tau(t)\le h_2-i\times\delta$, $i=0,1,2...,N-1$, we have $\alpha(t)\in[\frac{2N-i-1}{2N},\frac{2N-i}{2N}]$. Similar to Case 1, we obtain

$$\frac{1}{1-\alpha(t)}\ge-\frac{1}{\alpha(t)}\times\frac{(1-B_i)^2}{B_i^2}+\frac{1}{B_i^2}. \tag{18}$$

So $\phi - \frac{1}{\alpha(t)}\psi_1 - \frac{1}{1-\alpha(t)}\psi_2 < 0$, $\forall \alpha(t) \in [1-B_i, 1-A_i]$, if

$$\phi_i^2 = \phi - \gamma_i^2\psi_1 - \gamma_i^1\psi_2 < 0, \tag{19}$$

and

$$\phi - \frac{1}{1-B_i}\psi_1 - \frac{1}{B_i}\psi_2 < 0. \tag{20}$$

From the analysis of the two cases, note that if $\phi_i^1 < 0$ and $\phi_i^2 < 0$, we can get $\phi - 2\psi_1 - 2\psi_2 < 0$, which guarantees (17) and (20).

In conclusion, if $\phi_i^1 < 0$ and $\phi_i^2 < 0$, then $\dot{V}(t, x_t) < 0$. This completes the proof. ■

Now we will prove that Theorem 1 is less conservative than Theorem 1 in [13], which is rewritten as follows.

Theorem 2 ([13]). Given scalars $0 \le h_1 < h_2$, $0 \le \mu$, and positive integer $N \ge 1$, the system (1) with a time varying delay is asymptotically stable if there exist matrices $P > 0$, $Q_i > 0$, $i = 1,2,3$, $Z_j > 0$, $j = 1,2$ with appropriate dimensions such that the following LMIs hold.

$$\hat{\phi}_i^1 = \phi - \hat{\gamma}_i^1\psi_1 - \hat{\gamma}_i^2\psi_2 < 0, \tag{21}$$

and

$$\hat{\phi}_i^2 = \phi - \hat{\gamma}_i^2\psi_1 - \hat{\gamma}_i^1\psi_2 < 0, \tag{22}$$

where $i = 0,1,2..., N-1$, and

$$\hat{\gamma}_i^1 = \frac{2N}{i+1} + \frac{2N}{2N-(i+1)} - \frac{2N}{2N-i},$$

$$\gamma_i^2 = \frac{2N}{2N-i},$$

$$\phi = \begin{bmatrix} PA + A^T P + Q_1 + Q_2 + Q_3 - Z_1 & PA_1 & Z_1 & 0 \\ * & -(1-\mu)Q_3 & 0 & 0 \\ * & * & -Q_1 - Z_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix}$$

$$+[A \quad A_1 \quad 0 \quad 0]^T (h_1^2 Z_1 + h_{12}^2 Z_2)[A \quad A_1 \quad 0 \quad 0],$$

$$\psi_1 = [0 \quad -I \quad I \quad 0]^T Z_2 [0 \quad -I \quad I \quad 0],$$

$$\psi_2 = [0 \quad -I \quad 0 \quad I]^T Z_2 [0 \quad -I \quad 0 \quad I].$$

For Theorem 1 and Theorem 2, the following conclusion can be drawn.

Theorem 3. If there exist matrices $P > 0$, $Q_i > 0$, $i = 1,2,3$, $Z_j > 0$, $j = 1,2$ with appropriate dimensions such that (21)-(22)hold, then the matrices P, Q_i, $i = 1,2,3$, Z_j, $j = 1,2$, are feasible solutions to (5)-(6).

Proof. Suppose that there exist matrices $P > 0$, $Q_i > 0$, $i = 1,2,3$, $Z_j > 0$, $j = 1,2$ with appropriate dimensions such that (21)-(22) hold. Note that in Theorem 1, γ_i^1 can be rewritten as follows:

$$\gamma_i^1 = \left(\frac{2N}{i+1}\right)^2 - \frac{2N}{2N-i} \times \left(\frac{2N-i-1}{i+1}\right)^2$$

$$= \frac{2N}{i+1} + \frac{2N}{2N-(i+1)} - \frac{2N}{2N-i} + \left[\frac{2N}{2N-(i+1)} - \frac{2N}{2N-i}\right]\left\{\frac{[2N-(i+1)]^2}{(i+1)^2} - 1\right\}$$

$$= \hat{\gamma}_i^1 + \left[\frac{2N}{2N-(i+1)} - \frac{2N}{2N-i}\right]\left\{\frac{[2N-(i+1)]^2}{(i+1)^2} - 1\right\}$$

It is obvious that $\hat{\gamma}_i^1 \le \gamma_i^1$, so the matrices P, Q_i, $i = 1,2,3$, Z_j, $j = 1,2$ are feasible solutions to (5)-(6). ■

From Theorem 3, it is easy to see that by Lemma 2, the term $\left[\frac{2N}{2N-(i+1)} - \frac{2N}{2N-i}\right]\left\{\frac{[2N-(i+1)]^2}{(i+1)^2} - 1\right\}$ is introduced and thus Theorem 1 is less conservative than Theorem 2.

When the delay derivative is unknown or does not exist, the following result can be derived from Theorem 1 by setting $Q_3 = 0$.

Theorem 4. Given scalars $0 \le h_1 < h_2$, $0 \le \mu$, and positive integer $N \ge 1$, the system (1) with a time varying delay is asymptotically stable if there exist matrices $P > 0$, $Q_i > 0$, $i = 1,2$, $Z_j > 0$, $j = 1,2$ with appropriate dimensions such that the following LMIs hold.

$$\phi_i^1 = \phi - \gamma_i^1 \psi_1 - \gamma_i^2 \psi_2 < 0, \tag{23}$$

and

$$\phi_i^2 = \phi - \gamma_i^2 \psi_1 - \gamma_i^1 \psi_2 < 0, \tag{24}$$

where $i = 0,1,2\ldots, N-1$, and

$$\gamma_i^1 = \left(\frac{2N}{i+1}\right)^2 - \frac{2N}{2N-i} \times \left(\frac{2N-i-1}{i+1}\right)^2,$$

$$\gamma_i^2 = \frac{2N}{2N-i},$$

$$\phi = \begin{bmatrix} PA + A^T P + Q_1 + Q_2 - Z_1 & PA_1 & Z_1 & 0 \\ * & 0 & 0 & 0 \\ * & * & -Q_1 - Z_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix}$$

$$+[A \quad A_1 \quad 0 \quad 0]^T (h_1^2 Z_1 + h_{12}^2 Z_2)[A \quad A_1 \quad 0 \quad 0],$$

$$\psi_1 = [0 \quad -I \quad I \quad 0]^T Z_2 [0 \quad -I \quad I \quad 0],$$

$$\psi_2 = [0 \quad -I \quad 0 \quad I]^T Z_2 [0 \quad -I \quad 0 \quad I].$$

Remark 2. From Theorem 1 and Theorem 4, we can get the admissible upper bound h_2 of the time delay for a certain lower bound h_1 of the time delay by solving the following maximum problem.

Max h_2 subject to LMIs (5)-(6) in Theorem 1 or (23)-(24) in Theorem 4 respectively.

Note that Theorem 1 and Theorem 4 assume the existence of matrices P, Q and Z. If h_2 exceeds the admissible upper bound, matrices P, Q and Z do not exist, which means there is no solution to corresponding LMIs.

4. Numerical Examples

In this section, we use examples in Zhu et al. [13] to illustrate the advantages of the proposed stability results. The maximum problem in Remark 2 is solved and numerical results are obtained by using LMI SOLVER FEASP in MATLAB LMI Toolbox [14].

Example 1. Consider the system (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}.$$

In order to make a comparison, we employ the stability criteria given in [8], [12-13] and in Theorem 1 and 4 in this paper. When the derivative of delay is available, for the lower bound $h_1 = 0$, the admissible upper bounds h_2 of the time-varying delay are obtained for varies maximum delay derivative μ and the computational results are shown in Table 1. When the derivative of delay is unknown or doesn’t exist, for varies lower bound h_1, the admissible upper bounds h_2 are obtained and shown in Table 2. In Table 3, the computational complexity of the methods proposed in [8], [12], [13] and this paper are studied.

Table 1. Admissible upper bounds h_2 with varying μ and $h_1 = 0$

Method	μ	0.1	0.3	0.5	0.8	1
Shao[12]	h_2	5.463	2.216	1.127	0.871	0.871
Sun[8]	h_2	5.4764	2.2160	1.1272	0.8714	0.8714
Zhu[13](N=1)	h_2	5.466	2.236	1.154	0.929	0.929
Theorem 1(N=1)	h_2	5.466	2.236	1.154	0.929	0.929
Zhu[13](N=2)	h_2	5.469	2.26	1.179	0.98	0.98
Theorem 1(N=2)	h_2	5.475	2.278	1.202	1.005	1.005
Zhu[13](N=4)	h_2	5.478	2.285	1.208	1.02	1.02
Theorem 1(N=4)	h_2	5.494	2.307	1.233	1.044	1.044

Table 2. Admissible upper bounds h_2 with varying h_1 and unknown μ

Method	h_1	0.3	0.5	0.8	1
Shao[12]	h_2	1.0715	1.2191	1.4539	1.6169
Sun[8]	h_2	1.0716	1.2196	1.4552	1.6189
Zhu[13](N=1)	h_2	1.1232	1.2672	1.4974	1.6578
Theorem 4(N=1)	h_2	1.1232	1.2672	1.4974	1.6578
Zhu[13](N=2)	h_2	1.1677	1.3078	1.5333	1.6910
Theorem 4(N=2)	h_2	1.1907	1.3303	1.555	1.7124

Zhu[13](N=4)	h_2	1.2043	1.3429	1.5663	1.7228
Theorem 4(N=4)	h_2	1.2246	1.3619	1.5838	1.739

Table 3. A study of the computational complexity of the proposed method over the existing methods (when the derivative of the time delay is known)

Method	LMI	LMI Count	LMI size
Shao[12]	6	2	4×4
Sun[8]	26	2	9×9
Zhu[13]	6	$2\times N$	4×4
Theorem 1	6	$2\times N$	4×4

It can be seen from **Table 1** and **Table 2** that the stability results obtained in this paper are less conservative that those in [8], [12] and [13]. From **Table 3**, it can be seen that the reduced conservativeness does not bring about additional computational complexity compared with [13].

Example 2. Consider the system (1) with

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\quad A_1=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}$$

As with Example 1, here we calculate the admissible upper bound of the time delay that guarantees the asymptotic stability of system (1) using the methods proposed in [8], [12], [13], and Theorem 1 and Theorem 4 in this paper. For unknown maximum derivative μ of the time delay and varying lower bound h_1 of the time delay , the admissible upper bounds h_2 of the time-varying delay are obtained and shown in Table 4.

Table 4. Admissible upper bounds h_2 with varying h_1 and unknown μ

Method	h_1	1	2	3	4
Shao[12]	h_2	1.8737	2.5048	3.2591	4.0744
Sun[8]	h_2	1.9008	2.5663	3.3408	4.169
Zhu[13](N=1)	h_2	1.9422	2.5383	3.2749	4.0787
Theorem 4(N=1)	h_2	1.9422	2.5383	3.2749	4.0787
Zhu[13](N=2)	h_2	2.004	2.5650	3.2866	4.0818
Theorem 4(N=2)	h_2	2.0089	2.5829	3.2983	4.0848
Zhu[13](N=4)	h_2	2.0273	2.5915	3.3010	4.0855
Theorem 4(N=4)	h_2	2.0448	2.6051	3.3098	4.0877

For most of the cases, the obtained criteria have been shown to be less conservative than the existing ones in [8], [12-13]. Note that when the lower bound of the time delay is large (when h_1 =3 or 4 as shown in Table 4), the results obtained by Theorem 4 fail to compete with that of [8]. We intend to improve further the criteria in this paper to prevail completely the existing stability criteria for all the situations.

5. Conclusions

In this paper, delay-range-dependent stability criteria have been developed for a linear system with interval time-varying delay. By developing a new method to estimate the nonlinear time-varying coefficients derived from Jenson's integral inequality more tightly, less conservative stability criteria are derived by employing convex combination and delay partitioning method. The advantage of the criterion lies in its simplicity and less conservatism. Examples are also given to illustrate the reduced conservatism of the stability results.

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