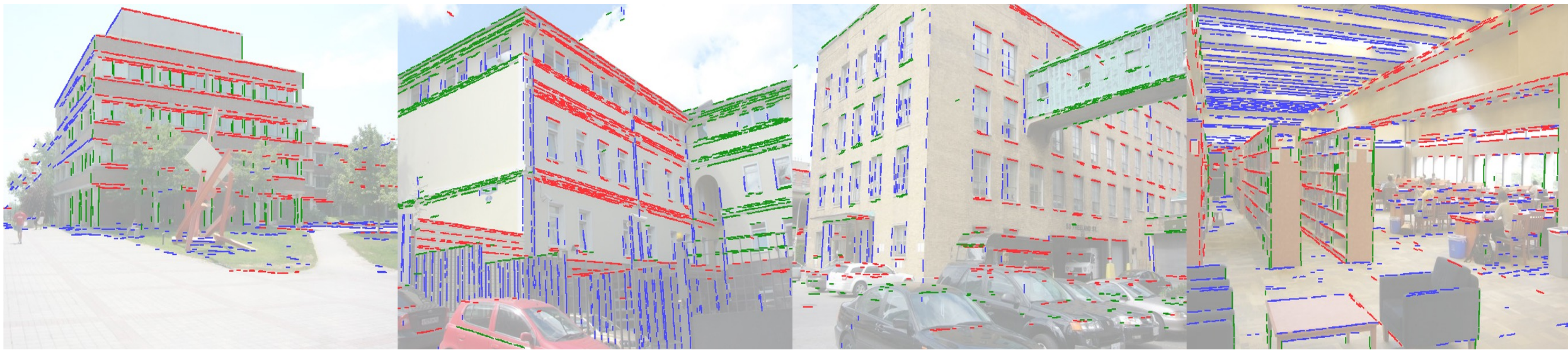
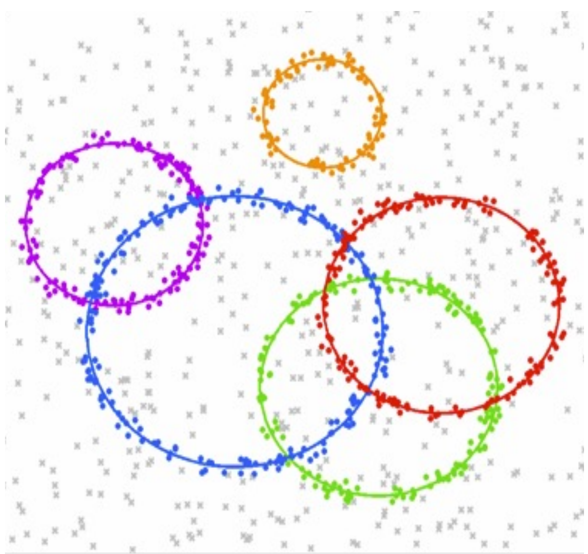
# Geometric Model Fitting





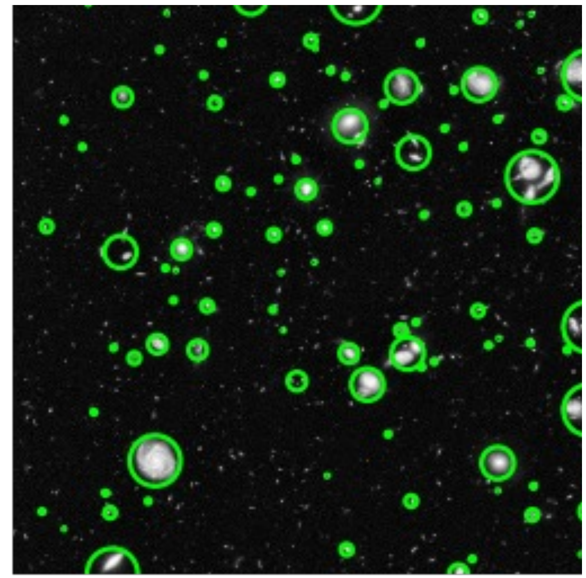
*with slides stolen from*
*Yuri Boykov, Steve Seitz and Rick Szeliski*

# Geometric Model Fitting

- Feature matching $(\mathbf{p}_i, \mathbf{p}'_i)$
- Model fitting (e.g. homography estimation for panoramas)
  - How many points to choose?
  - Least square model fitting
  - RANSAC (robust method for model fitting)
- Multi-model fitting problems

# Flashbacks: feature **detectors**

**Harris corners**



**Dog**

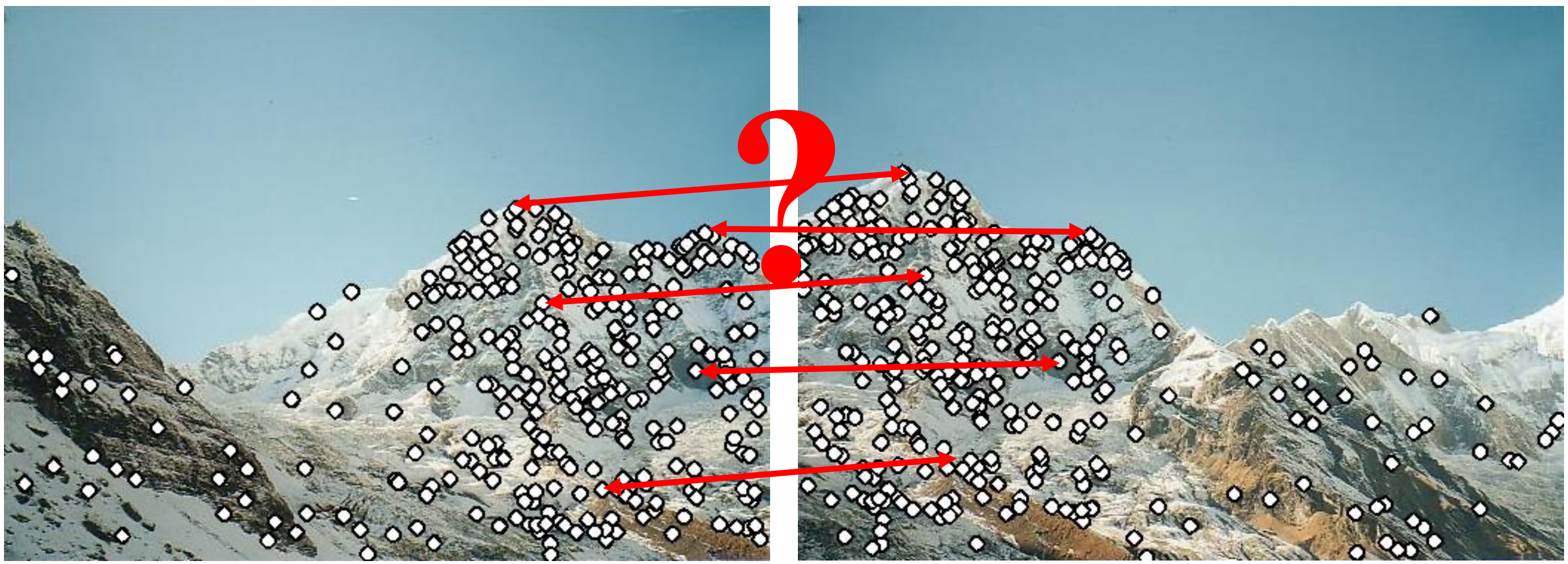
python code from “FeaturePoints.ipynb”

```
from skimage.feature import corner_harris, corner_subpix, corner_peaks

hc_filter = corner_harris(image_gray)
peaks = corner_peaks(hc_filter)
```

```
from skimage.feature import blob_dog

blobs = blob_dog(image_gray)
```

# Flashbacks: feature **descriptors**

We know how to detect points
Next question: **How to match them?**




need **point descriptors** that should be

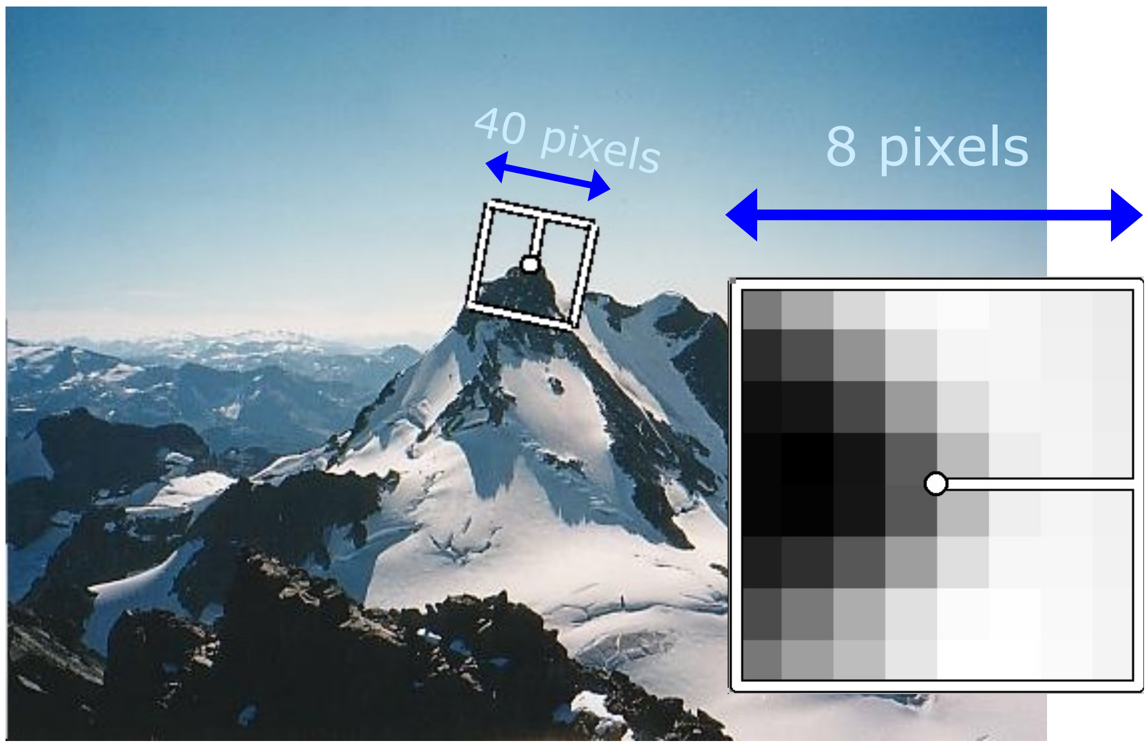
- Invariant (e.g. to gain/bias, rotation, projection, etc)
- Distinctive (to avoid false matches)

# Flashbacks: MOPS descriptor

8x8 oriented patch

- Sampled at 5 x scale

Bias/gain normalization: $I' = (I - \mu)/\sigma$



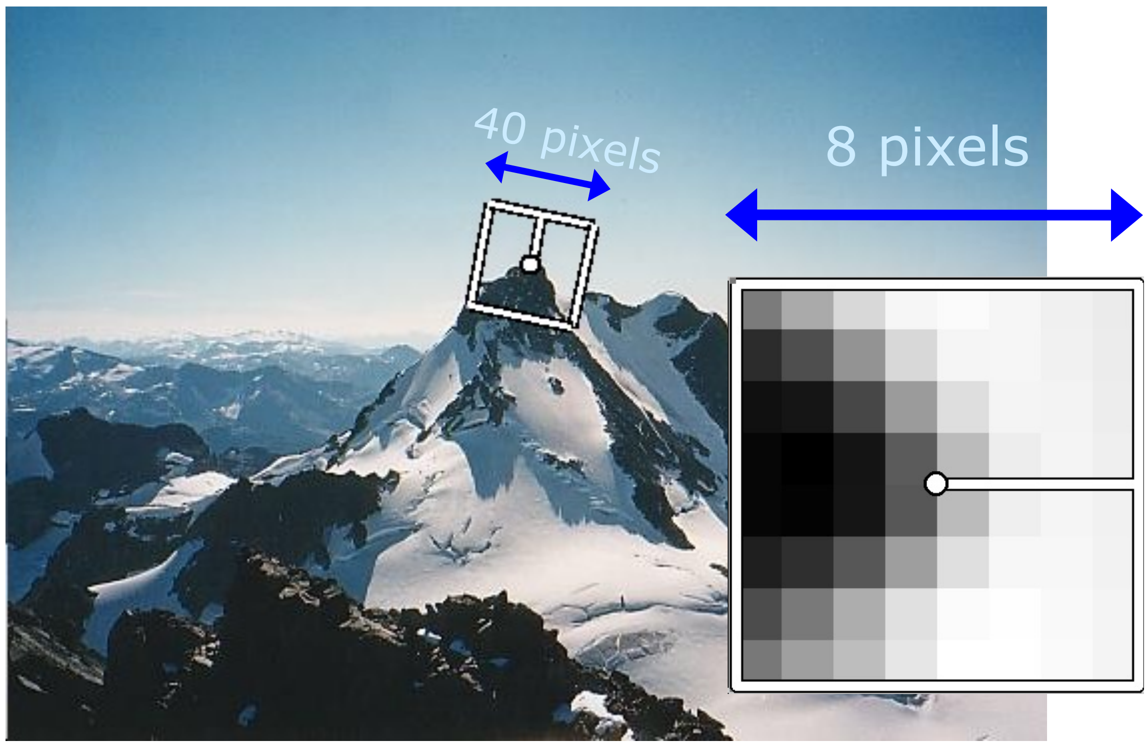

Another popular idea (SIFT): use gradient orientations inside the patch as a descriptor (also invariant to gain/bias)

# Flashbacks: MOPS descriptor

8x8 oriented patch

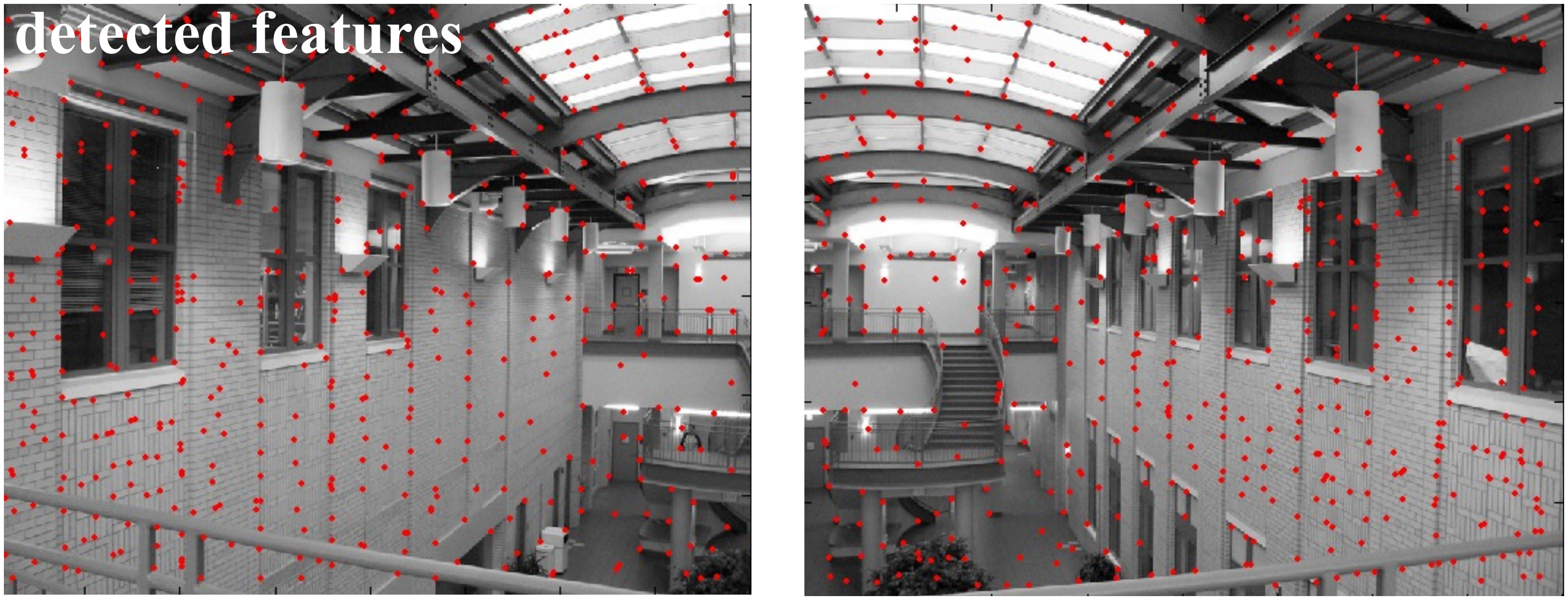
- Sampled at 5 x scale

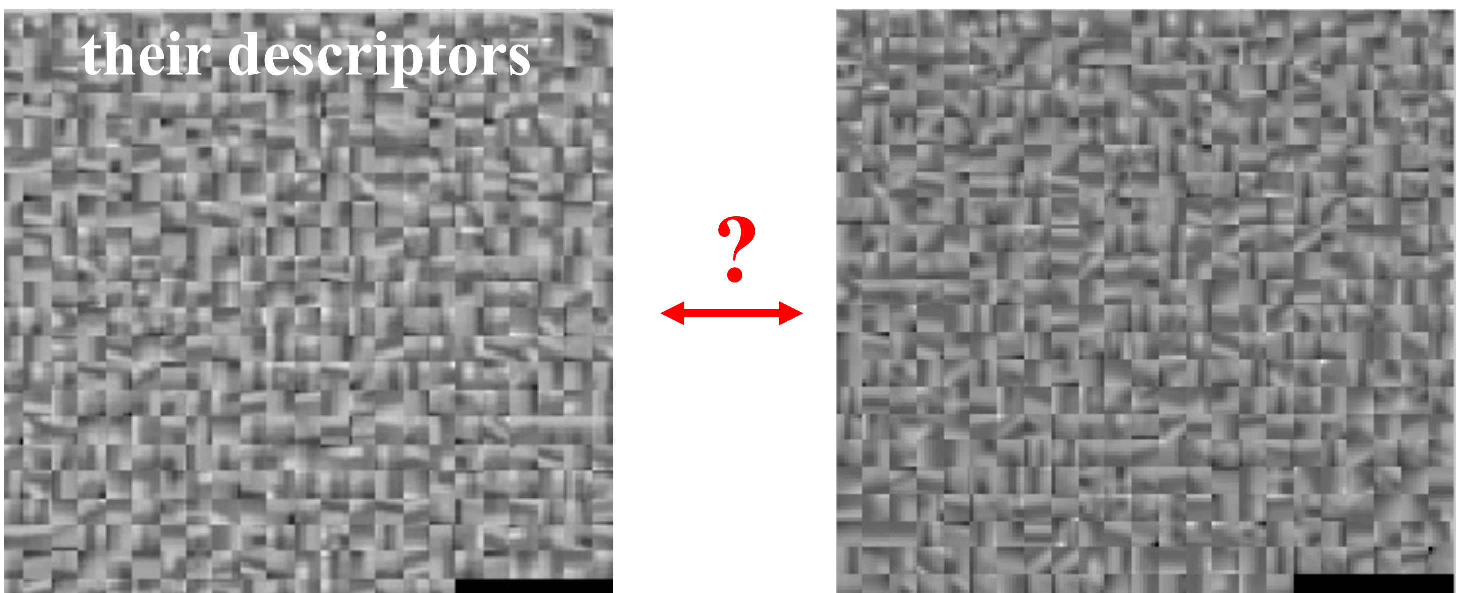
Bias/gain normalization: $I' = (I - \mu)/\sigma$




Popular descriptors: MOPS, SIFT, SURF, HOG, BRIEF, many more…

# Feature matching



# Feature matching

## Optimal matching:

- Bipartite matching, quadratic assignment (QA) problems
  - too expensive

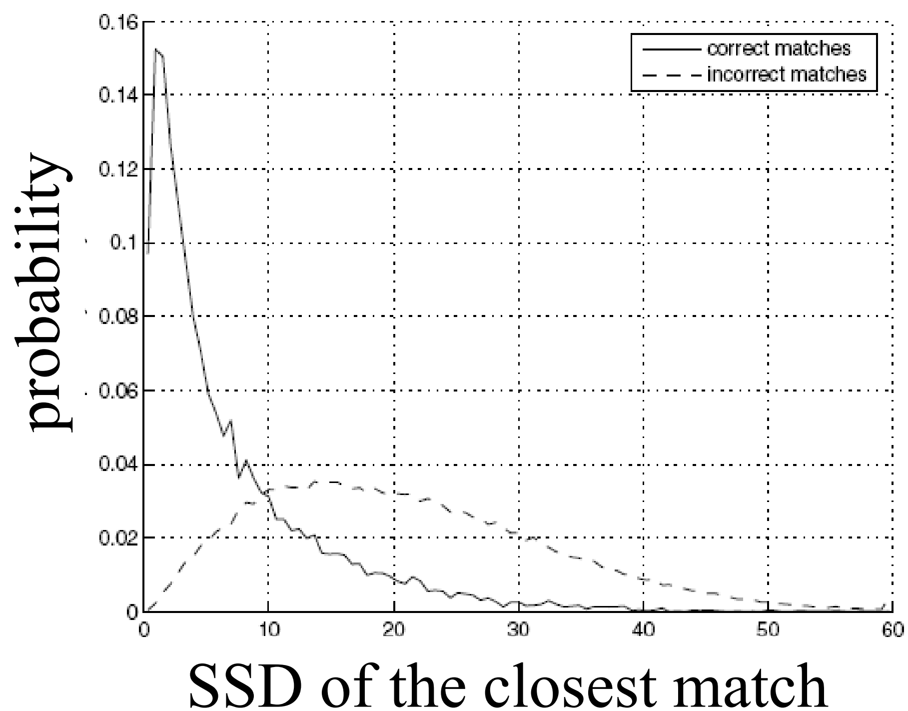
## Common simple approach:

- use SSD (sum of squared differences) between two descriptors (patches).
- for each feature in image 1 find a feature in image 2 with the lowest SSD
- accept a match if SSD(patch1,patch2) < T (threshold)

# Feature matching

SSD(patch1,patch2) < T

How to set threshold T?

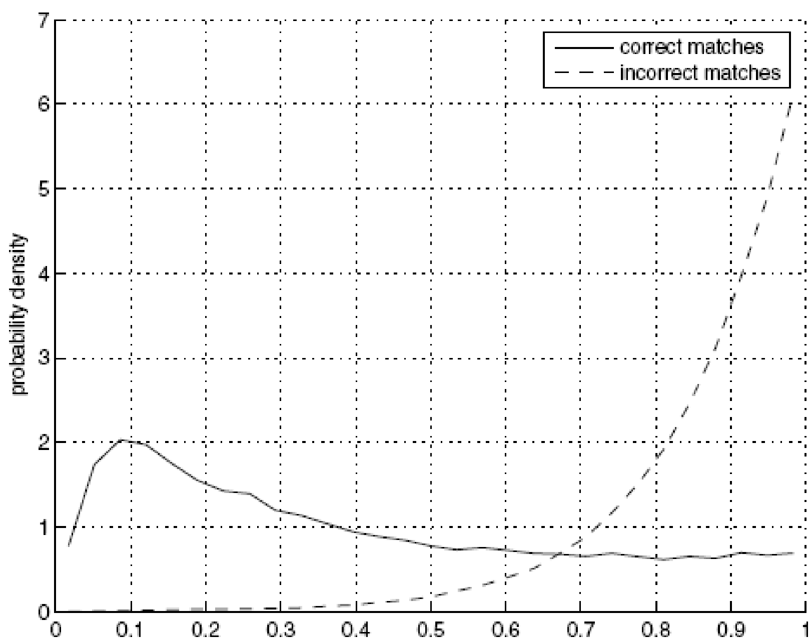



**no threshold** T **is good for separating correct and incorrect matches**

# Feature matching

A better way [Lowe, 1999]:

- SSD of the closest match (SSD1)
- SSD of the <u>second-closest</u> match (SSD2)
- Accept the best match if it is much better than the second-best match (and the rest of the matches)




(SSD1) / (SSD2)

**easier to select threshold T for decision test**

(SSD1) / (SSD2) < T

# Python example (BRIEF descriptor)

```
from skimage.feature import (corner_harris, corner_peaks, plot_matches, BRIEF, match_descriptors)

keypointsL = corner_peaks(corner_harris(imL), threshold_rel=0.0005, min_distance=5)
keypointsR = corner_peaks(corner_harris(imR), threshold_rel=0.0005, min_distance=5)

extractor = BRIEF()

extractor.extract(imL, keypointsL)
keypointsL = keypointsL[extractor.mask]
descriptorsL = extractor.descriptors

extractor.extract(imR, keypointsR)
keypointsR = keypointsR[extractor.mask]
descriptorsR = extractor.descriptors

matchesLR = match_descriptors(descriptorsL, descriptorsR, cross_check=True)
```

**find the closest match p' for any feature p**

**crosscheck:** keep pair (p,p') only if p is the best match for p'

# How to fit a homorgaphy???

**What problems do you see for homography estimation?**

# How to fit a homorgaphy???

**What problems do you see for homography estimation?**

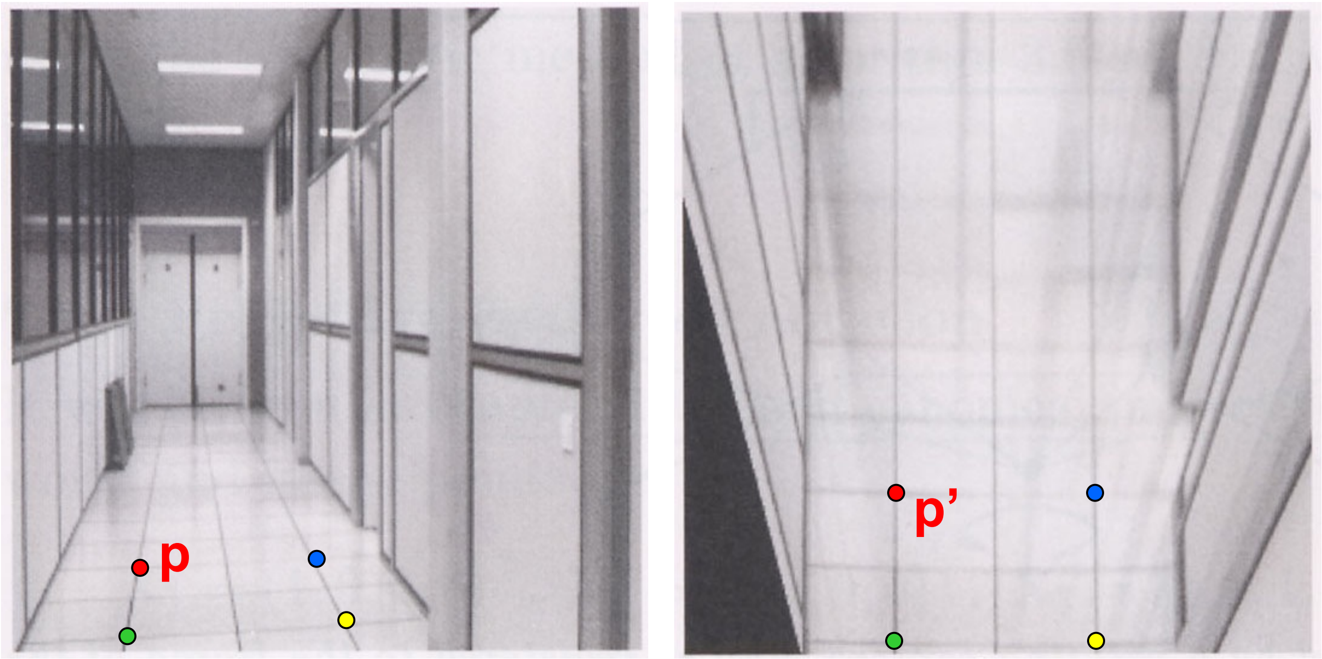
**Issue 1:** the number of matches $(\mathbf{p}_i, \mathbf{p'}_i)$ is more than 4

**Answer:** model fitting via “least squares” (later, slide 21)

**Issue 2:** too many *outliers* or wrong matches $(\mathbf{p}_i, \mathbf{p'}_i)$

**Answer:** robust model fitting via RANSAC (later, slide 35)

# Recall: Homography from 4 points



# Recall: Homography from 4 points

Consider one match (point-correspondence) $p = (x, y) \quad \rightarrow \quad p' = (x', y')$

$$\mathbf{p}' = \mathbf{H}\mathbf{p} \qquad \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

**After eliminating** $w = gx + hy + i$ **:**

$$\Rightarrow \qquad \begin{aligned} ax + by + c - gxx' - hyx' - ix' &= 0 \\ dx + ey + f - gxy' - hyy' - iy' &= 0 \end{aligned}$$

**Two equations linear w.r.t unknown coefficients of matrix H and quadratic w.r.t. known point coordinates (x,y,x',y')**

# Recall: Homography from 4 points

Consider 4 point-correspondences $p_i = (x_{i,} y_i) \;\rightarrow\; p'_i = (x'_{i,}\, y'_i)$

$$\mathbf{p}'_i = \mathbf{H}\mathbf{p}_i \qquad \begin{bmatrix} w_i x'_i \\ w_i y'_i \\ w_i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

**for i=1,2,3,4**

$$\Rightarrow \begin{array}{l} ax_i + by_i + c - gx_i x'_i - hy_i x'_i - ix'_i = 0 \\ dx_i + ey_i + f - gx_i y'_i - hy_i y'_i - iy'_i = 0 \end{array}$$

**Special case of DLT method (see p.89 in Hartley and Zisserman)**

**Can be written as matrix multiplication** $\mathbf{A}_i \cdot \mathbf{h} = \mathbf{0}$ **for i=1,2,3,4**

**where** $\mathbf{h} = [\,a\; b\; c\; d\;\; e\;\; f\;\; g\;\; h\;\; i\,]^T$ **is a vector of unknown coefficients in** $\mathbf{H}$

**and** $\mathbf{A}_i$ **is a 2x9 matrix based on known point coordinates** $x_i, y_i, x'_i, y'_i$

# Recall: Homography from 4 points

Consider 4 point-correspondences $p_i = (x_i, y_i) \rightarrow p'_i = (x'_i, y'_i)$

$$\mathbf{p}'_i = \mathbf{H}\mathbf{p}_i \quad \Rightarrow \quad \underset{2\text{x}9}{\mathbf{A}_i} \cdot \underset{9\text{x}1}{\mathbf{h}} = \underset{2\text{x}1}{\mathbf{0}} \quad \text{for i=1,2,3,4}$$

**All four matrix equations can be "stacked up" as**

$$\underset{8\text{x}9}{\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \\ \mathbf{A}_4 \end{bmatrix}} \cdot \mathbf{h} = \underset{8\text{x}1}{\mathbf{0}}$$

**or**

$$\boxed{\underset{8\text{x}9}{\mathbf{A}} \cdot \underset{9\text{x}1}{\mathbf{h}} = \underset{8\text{x}1}{\mathbf{0}}}$$

**Q:** how many solutions for h? A: none B: one C: many

# Recall: Homography from 4 points

Consider 4 point-correspondences $p_i = (x_i, y_i) \rightarrow p'_i = (x'_i, y'_i)$

$$\mathbf{p}'_i = \mathbf{H}\mathbf{p}_i \qquad \Rightarrow \qquad \boxed{\underset{8\text{x}9}{\mathbf{A}} \cdot \underset{9\text{x}1}{\mathbf{h}} = \underset{8\text{x}1}{\mathbf{0}}} \qquad (*)$$

for i=1,2,3,4

**8 linear equations, 9 unknowns: trivial solution h=0?**

**All solutions h form the (right) null space of A of dimension 1, but they represent the same transformation (as homographies can be scaled)**

**To find one specific solution h, for now fix one element,** *e.g.* $\boldsymbol{i = 1}$

as discussed in topic 7, **this may not work** more generally, should fix norm $\|\mathbf{h}\|=1$ (later)

$$\Rightarrow \qquad \boxed{\underset{8\text{x}8}{\mathbf{A}_{1:8}} \cdot \underset{8\text{x}1}{\mathbf{h}_{1:8}} = \underset{8\text{x}1}{-\mathbf{A}_9}}$$

**first 8 columns of A** **first 8 rows of h** **9th columns of A**

# Recall: Homography from 4 points

Consider 4 point correspondences $p_i = (x_i, y_i) \rightarrow p'_i = (x'_i, y'_i)$

$$\mathbf{p}'_i = \mathbf{H}\mathbf{p}_i \quad \Rightarrow \quad \boxed{\mathbf{A}_{1:8} \cdot \mathbf{h}_{1:8} = -\mathbf{A}_9}$$

for i=1,2,3,4

8x8 8x1 8x1

# More than 4 points

Consider 4 point correspondences $p_i = (x_i, y_i) \rightarrow p'_i = (x'_i, y'_i)$

$$\mathbf{p'}_i = \mathbf{H}\mathbf{p}_i \quad \Rightarrow \quad \boxed{\mathbf{A}_{1:8} \cdot \mathbf{h}_{1:8} = -\mathbf{A}_9}$$

for i=1,2,3,4

8x8 8x1 8x1

**Questions:**

**What if 4 points correspondences are known with error?**

**Are there any benefits from knowing more point correspondences?**

First, consider a simpler model fitting problem…

# Simpler example: line fitting

Assume a set of data points $(X_1, X'_1)$, $(X_2, X'_2)$, $(X_3, X'_3)$, ...

(e.g. person's height vs. weight)

We want to fit a model (e.g. a line) to predict $X'$ from $X$

$$a \cdot X + b = X'$$

How many pairs $(X_i, X'_i)$ do we need to find $a$ and $b$?

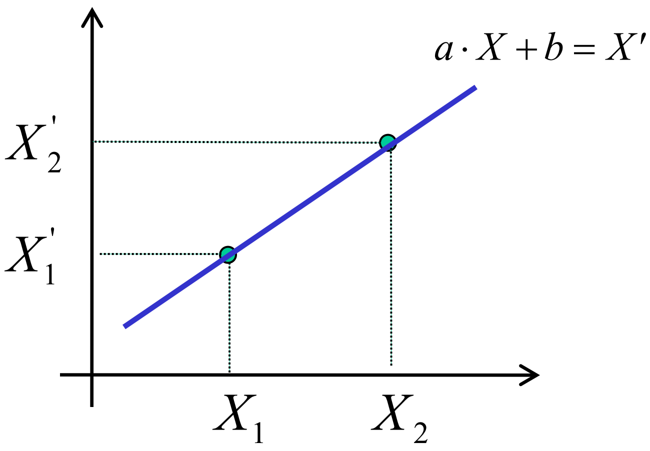
$$X_1 a + b = X'_1$$

$$X_2 a + b = X'_2$$

$$\begin{bmatrix} X_1 & 1 \\ X_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} X'_1 \\ X'_2 \end{bmatrix}$$

$$\boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{B}$$

$$\boxed{x = A^{-1} \cdot B}$$



# Simpler example: line fitting

Assume a set of data points $(X_1, X_1'), (X_2, X_2'), (X_3, X_3'), \ldots$

(e.g. person's height vs. weight)

We want to fit a model (e.g. a line) to predict $X'$ from $X$

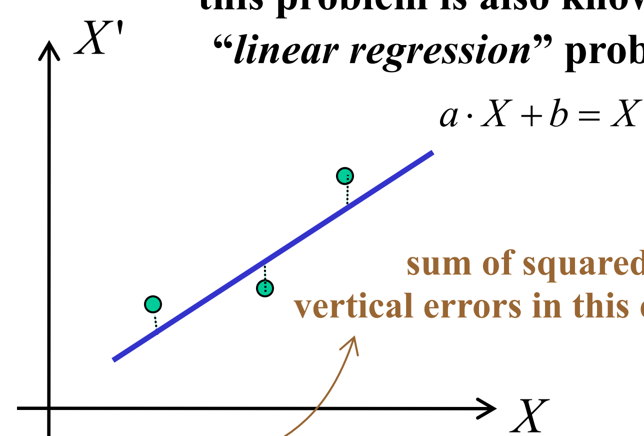
$$a \cdot X + b = X'$$

What if the data points $(X_i, X_i')$ are noisy?

over-constrained

$$\underbrace{\begin{bmatrix} X_1 & 1 \\ X_2 & 1 \\ X_3 & 1 \\ \ldots & \ldots \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} X_1' \\ X_2' \\ X_3' \\ \ldots \end{bmatrix}}_{B}$$

**this problem is also known as "*linear regression*" problem**




**sum of squared vertical errors in this example**

$$\min_x \underbrace{\|Ax - B\|^2}$$ **(least-squares)**

$$\boxed{x = A^{-1} \cdot B}$$

**where** $A^{-1} \equiv V \cdot W^{-1} \cdot U^T$ **is a *pseudo-inverse*** **based on SVD decomposition** $A = U \cdot W \cdot V^T$

(in python, one can use *svd* function in library *numpy.linalg*)

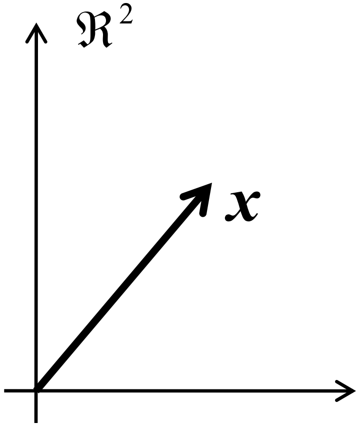
# SVD: rough idea

$$\underset{\textit{embed}}{}\quad\underset{\textit{scale}}{}\quad\underset{\textit{rotate}}{}$$

$$A = \overset{\textit{embed}}{U} \cdot \overset{\textit{scale}}{W} \cdot \overset{\textit{rotate}}{V^T}$$

**M ≥ N:** **MxN** **MxN** **NxN** **NxN**

**where $U$ and $V$ are matrices with ortho-normal columns and $W$ is diagonal with elements $w_i \geq 0$**
(see "Numerical Recipes in C", edition 2, Sec. 2.6)

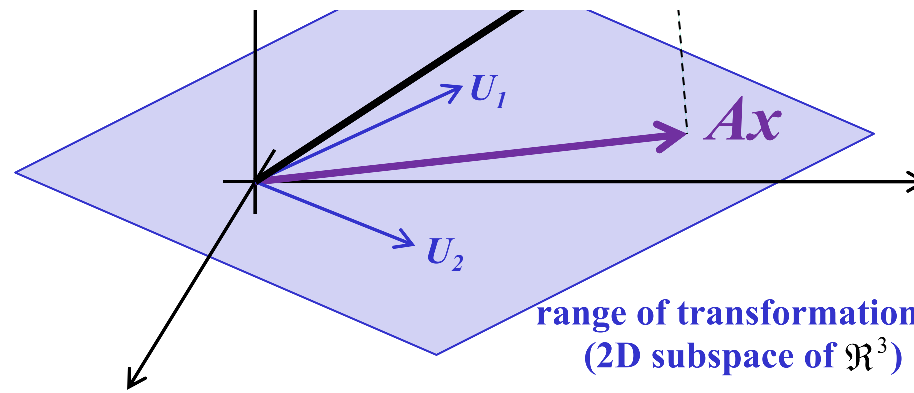



$$\underset{3\text{x}2}{A} \cdot x$$

$\Re^3$

**Q: where are the points from R² mapped to?**

A: point B: line C: plane D: whole R³




**range of transformation $A$ (2D subspace of $\Re^3$)**

$U_i$ (column-vectors of $U$) form the basis of this subspace

**How does SVD help to solve *least-squares* ?**

$$\min_x \|Ax - B\|^2$$

**projection of $B$ onto range of $A$**

$$\boxed{x = A^{-1} \cdot B} \equiv V \cdot W^{-1} \cdot U^T \cdot B$$

**Equivalent (fast to compute) expression**

$$\underset{\text{Nx1}}{x} = \underset{\text{NxN}}{(A^TA)^{-1}} \cdot \underset{\text{NxM}}{A^T} \cdot \underset{\text{Mx1}}{B}$$

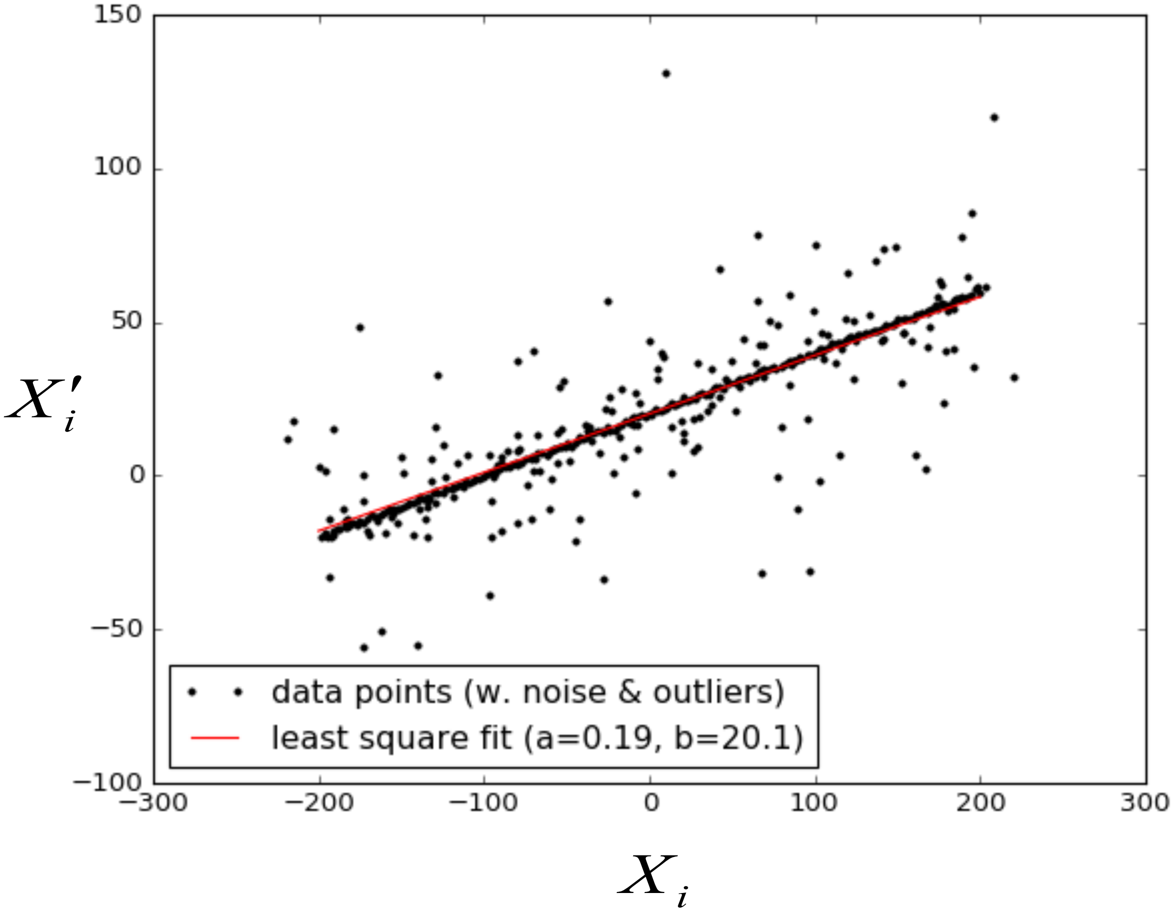
**Indeed:** $A^TA = VW\cancel{U^TU}WV^T = VW^2V^T$

**so** $(A^TA)^{-1} \cdot A^T = VW^{-2}\cancel{V^T \cdot V}WU^T = VW^{-1}U^T$

**If M>>N computing inverse of *positive semi-definite* NxN matrix $A^TA$ can be faster than SVD of MxN matrix A**

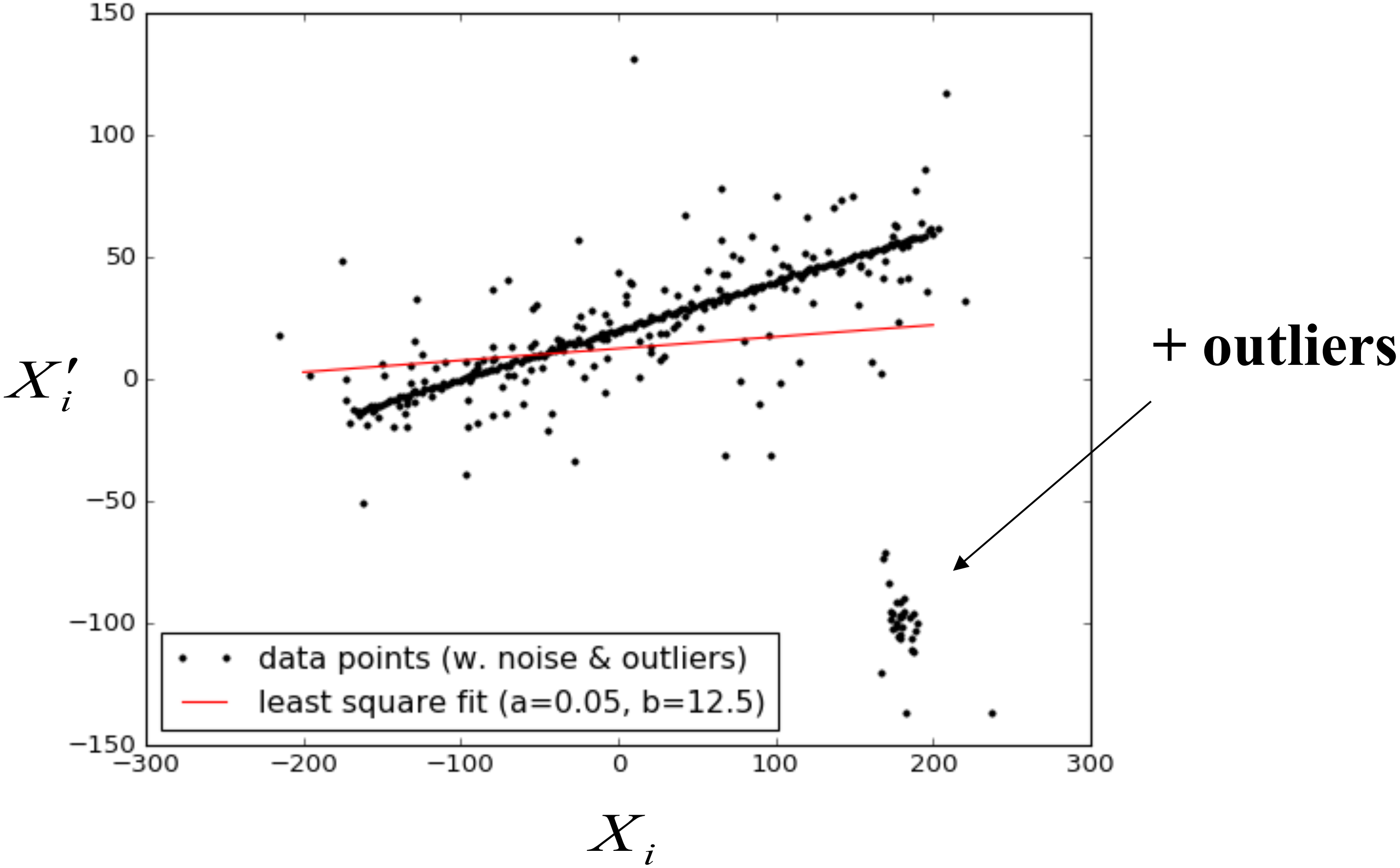
# Least squares line fitting

Data generated as $X_i' = a\ X_i + b + \delta X_i$ for $a=0.2,\ b=20$ and Normal noise $\delta X_i$



# Least squares fail in presence of outliers

Data generated as $X'_i = a\ X_i + b + \delta X_i$ for $a$=0.2, $b$= 20 and Normal noise $\delta X_i$




**For cases with outliers we need a more robust method (e.g. RANSAC, coming soon)**

# Model fitting robust to outliers

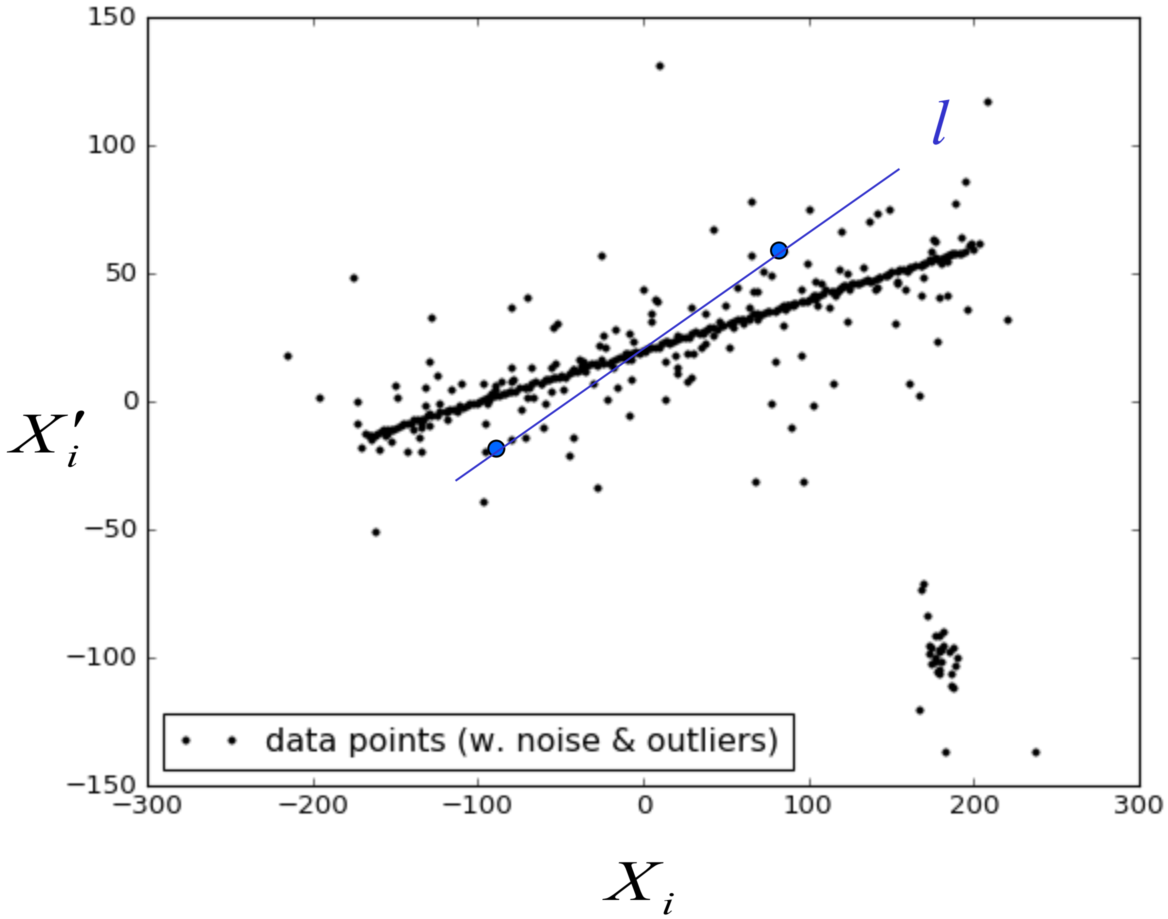
We need a method that can separate inliers from outlliers

**RANSAC**

***ran**dom **sa**mpling*
***c**onsensus*

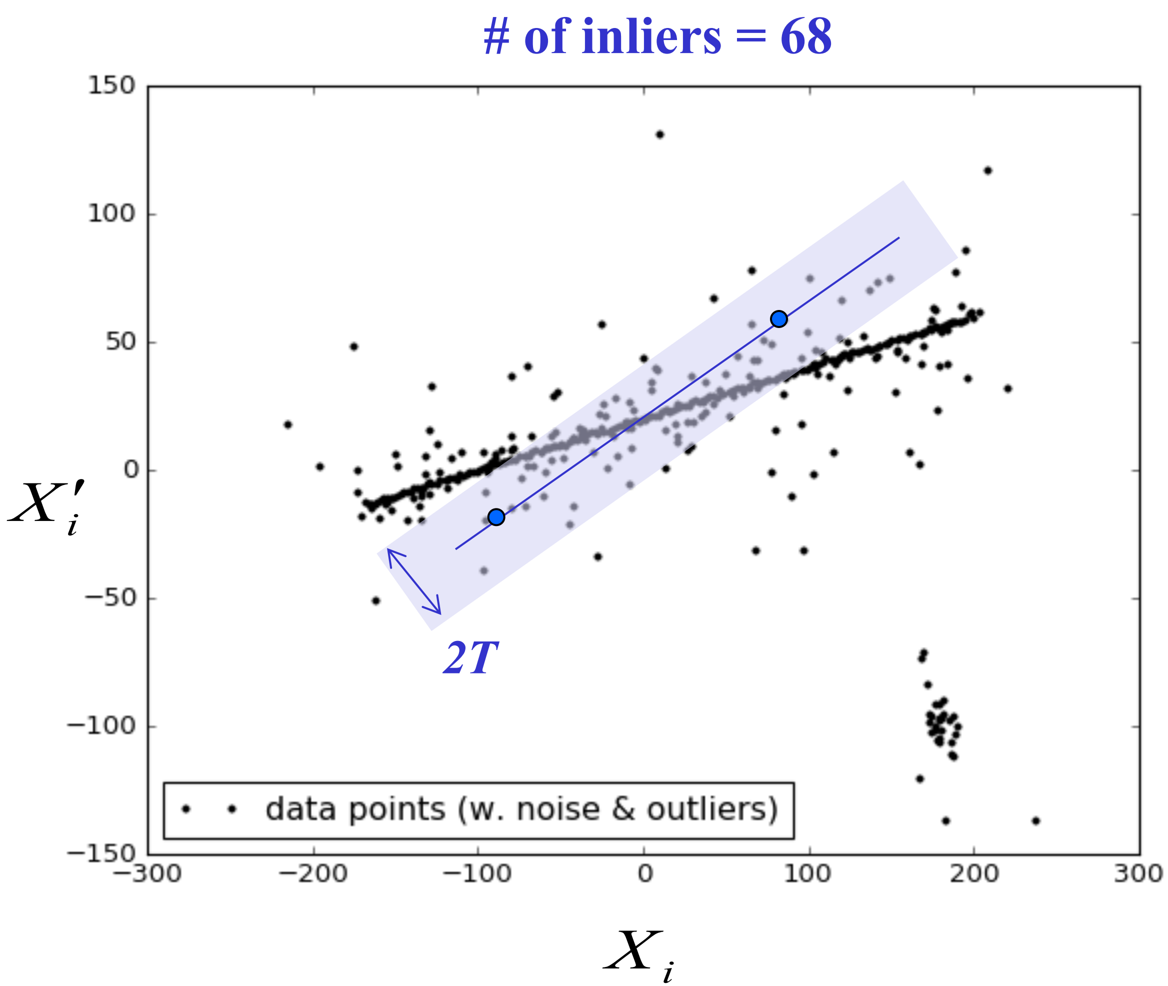
**[Fischler and Bolles, 1981]**

# RANSAC (for line fitting example)




1. **randomly sample two points from the set, get a line**

# RANSAC (for line fitting example)

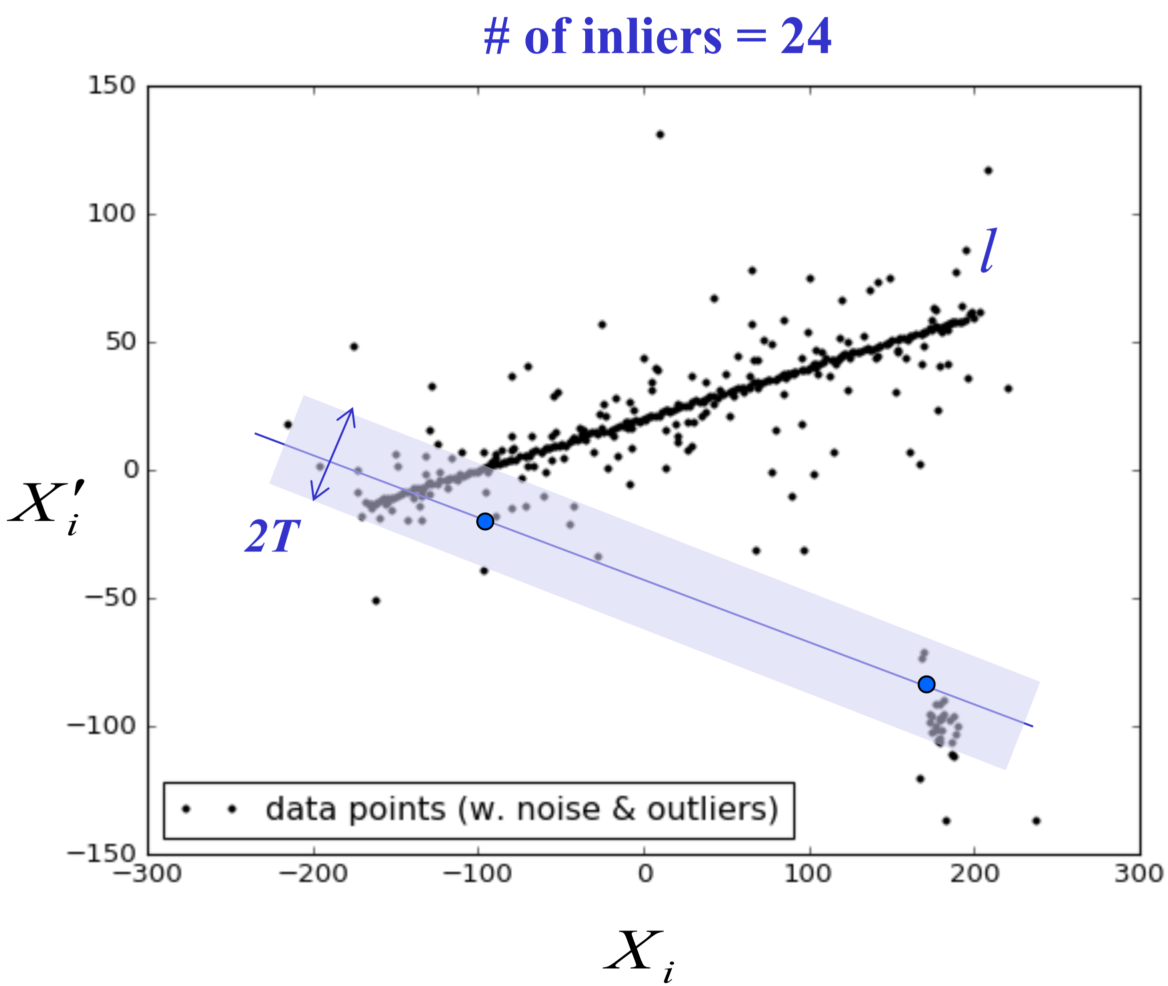



1. **randomly sample two points from the set, get a line**

2. **count inliers $p$ for <u>threshold</u> $T$**

$$\| p - l \| \leq T$$

# RANSAC (for line fitting example)



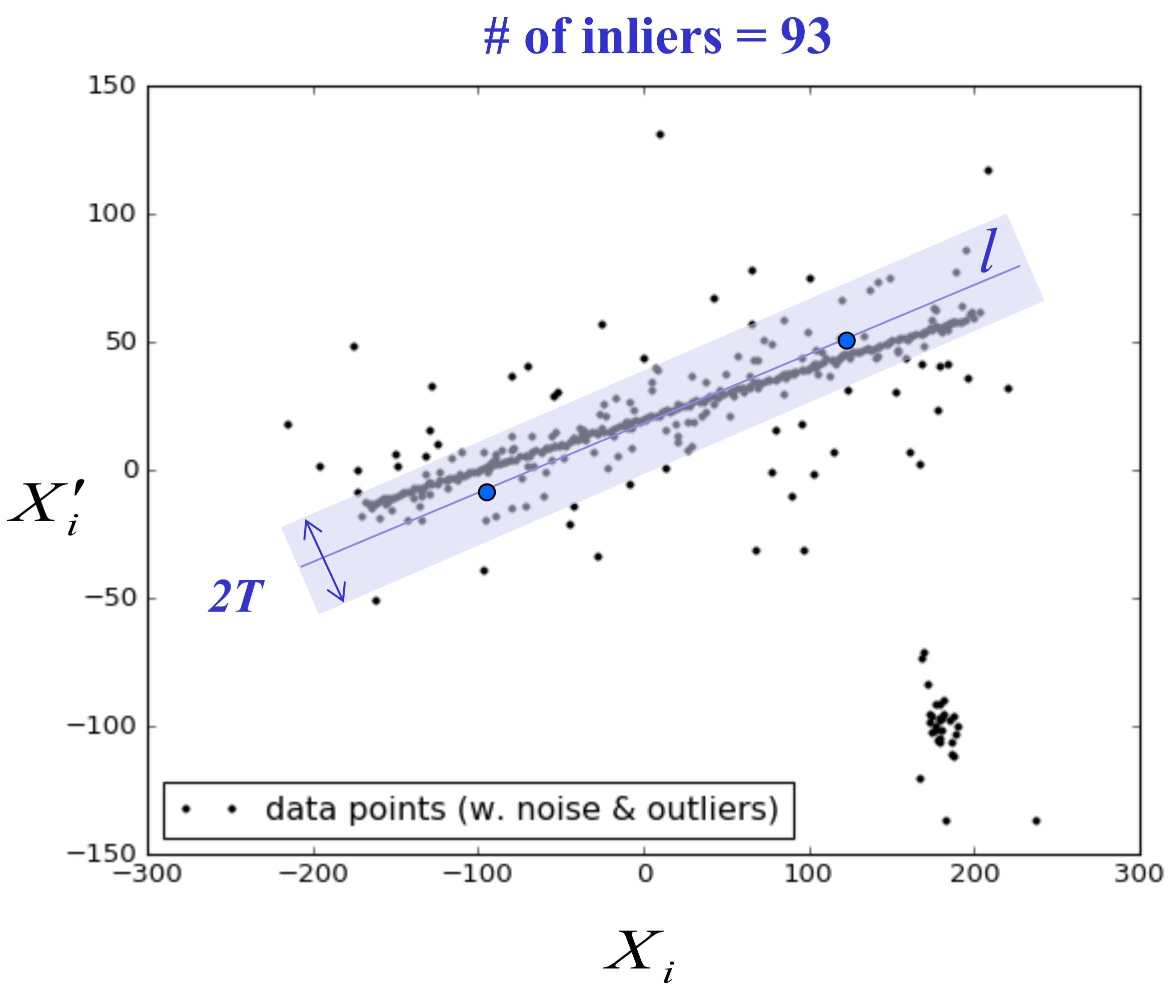

1. randomly sample **two points** from the set, get a **line**

2. count **inliers** $p$ for **threshold** $T$

$$\| p - l \| \leq T$$

3. repeat

# RANSAC (for line fitting example)



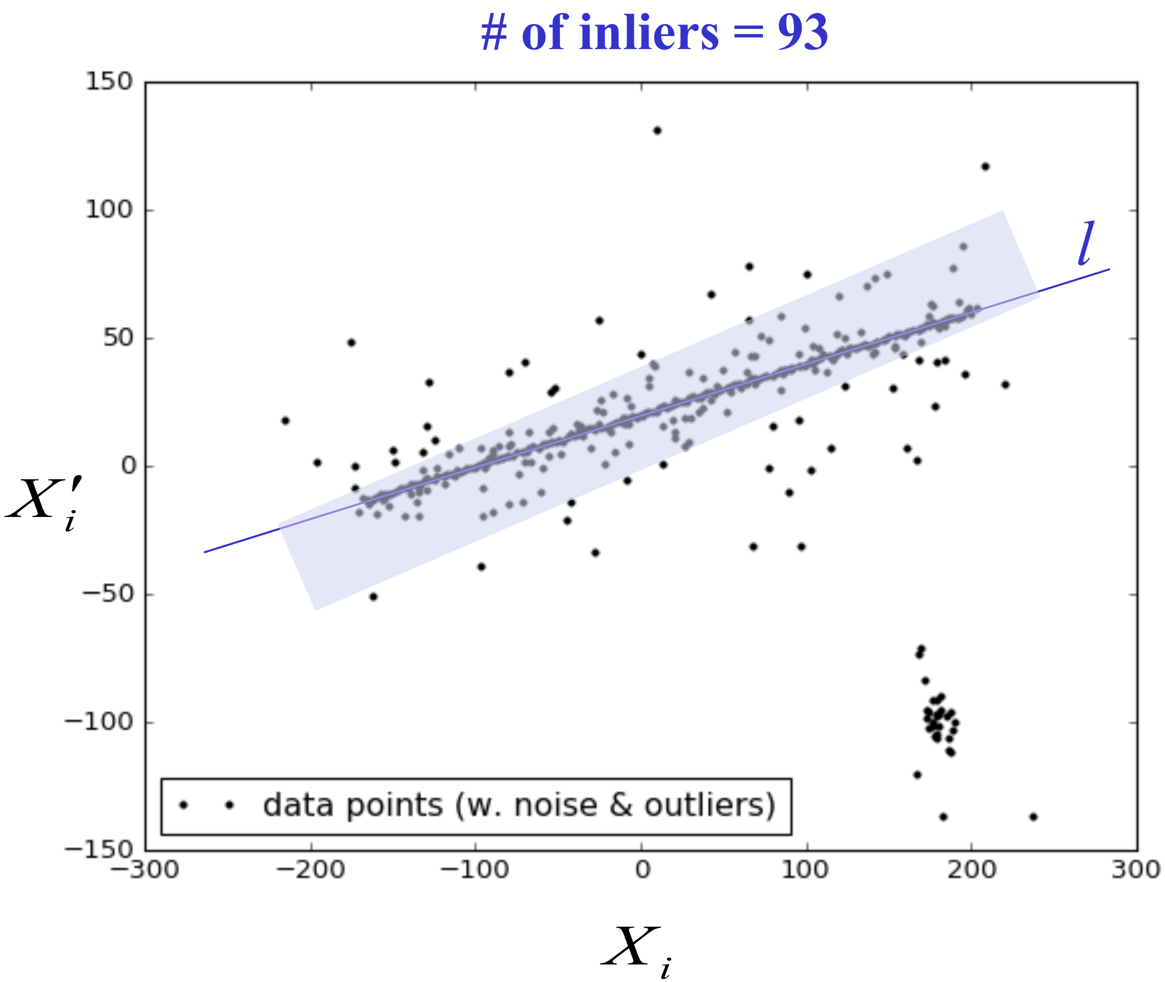

1. randomly sample **two points** from the set, get a **line**

2. count **inliers** $p$ for **threshold** $T$

$$\| p - l \| \leq T$$

3. repeat N times and select model with **most inliers**

# RANSAC (for line fitting example)




1. **randomly sample two points from the set, get a line**

2. **count inliers $p$ for threshold $T$**

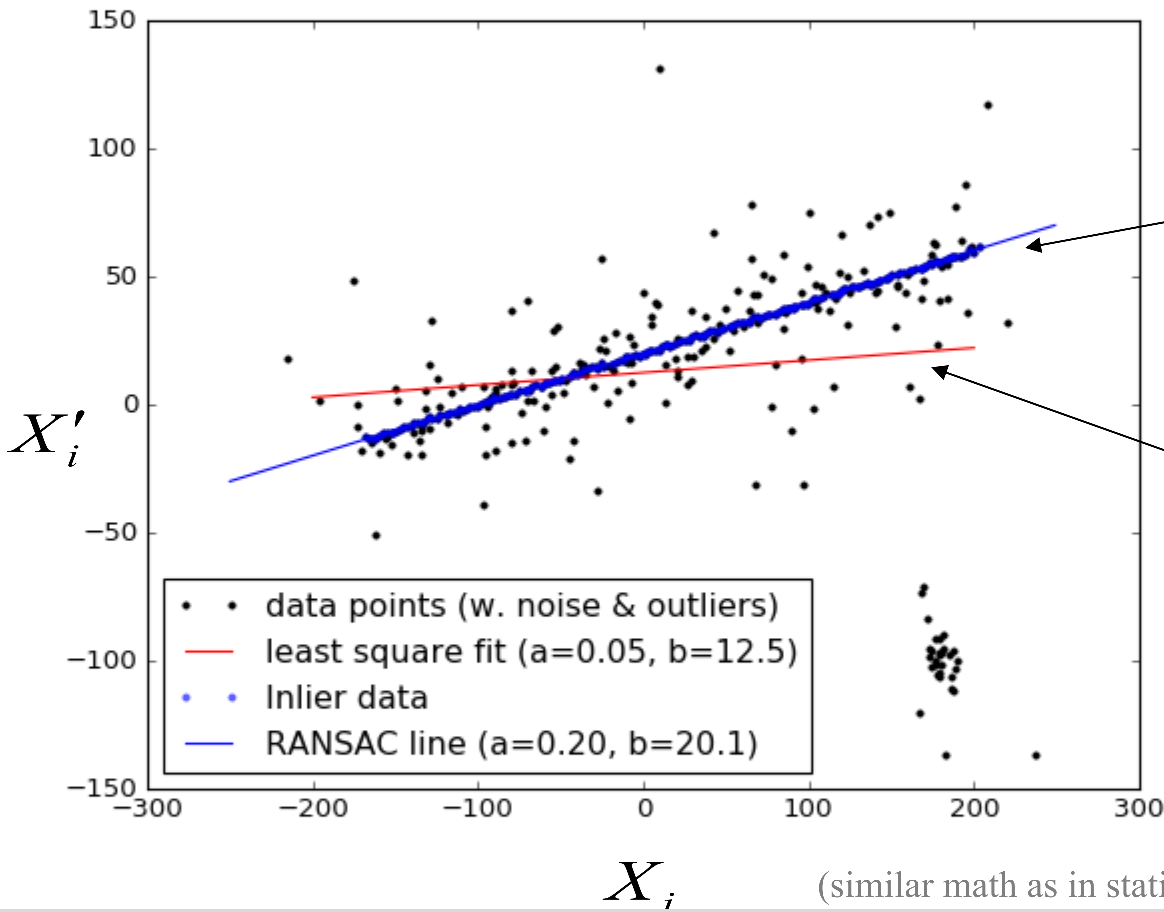
$$\| p - l \| \le T$$

3. **repeat N times and select model with most inliers**

4. **Use *least squares* to fit a model (line) to this largest set of inliers**

**Q: Assume know percentage of outliers in the data.**
**How many pairs of points (N) should be sampled to have high confidence** (e.g. 95%) **that at least one pair are both inliers?** [Fischler and Bolles, 1981]

# RANSAC (for line fitting example)



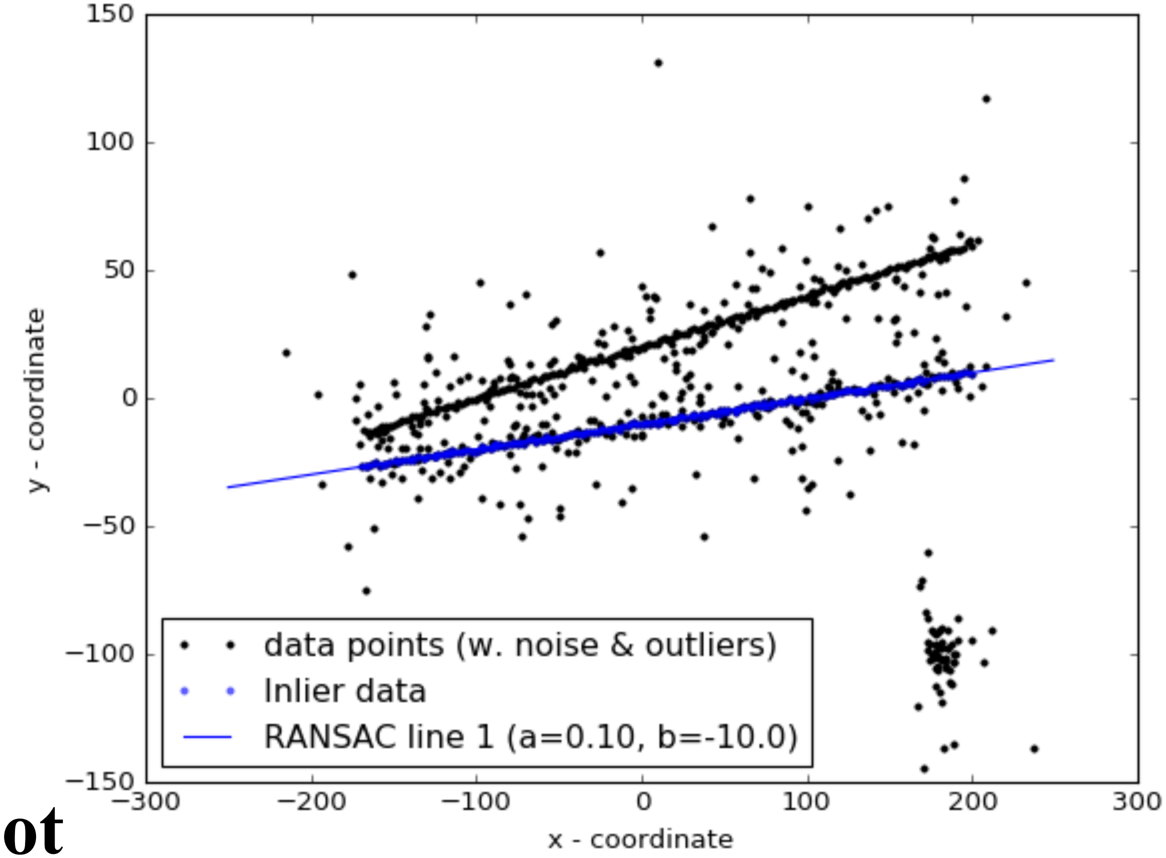

**line model reliably estimated via RANSAC with only N=10 samples**
(*least squares* fit to the largest set of inliers)

***least squares*** **line model fit to all points**

(similar math as in statistical analysist of RANSAC success rate)

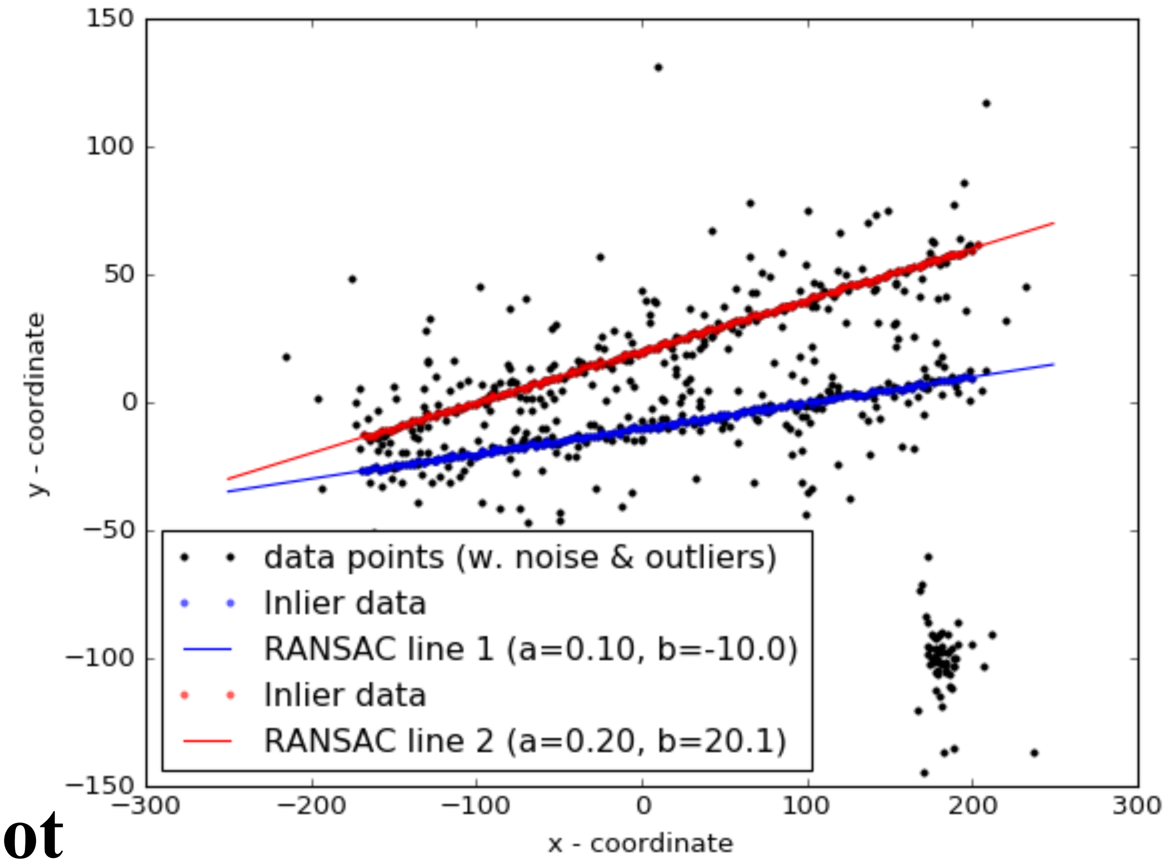
***Birthday Paradox***: in a group of random 23 people the probability that at least two have same birthday is 50.7%

# So, how do we find multiple models?




**Why not RANSAC again?**

# So, how do we find multiple models?




**Why not**
**RANSAC**
**again?**

remove inliers for line 1
and use RANSAC again
(**sequential RANSAC**)

# Homography from N ≥ 4 points

**Consider N point correspondences** $p_i = (x_i, y_i) \rightarrow p'_i = (x'_i, y'_i)$

$$\mathbf{p}'_i = \mathbf{H}\mathbf{p}_i \qquad \Rightarrow \qquad \boxed{\underset{2Nx9}{\mathbf{A}} \cdot \underset{9x1}{\mathbf{h}} = \underset{2Nx1}{\mathbf{0}}} \qquad (*)$$

for $i$ =1,…,N

over-constrained system

*Approach 1*: **add constraint *i* =1**. So, there are only 8 unknowns.

Set up a system of linear equations for vector of unknowns $\boldsymbol{h}_{1:8}$ =[a,b,c,d,e,f,g,h]$^T$

$$\underset{2Nx8}{\mathbf{A}_{1:8}} \cdot \underset{8x1}{\mathbf{h}_{1:8}} = \mathbf{B} = -\underset{2Nx1}{\mathbf{A}_9}$$

solve $\boxed{\min_{h_{1:8}} \| A_{1:8} h_{1:8} - B \|^2}$ (*least-squares*)

compute inverse for $\mathbf{A}_{1:8}^T \mathbf{A}_{1:8}$ as in line fitting, then $\boldsymbol{h}_{1:8} = (\mathbf{A}_{1:8}{}^T \cdot \mathbf{A}_{1:8})^{-1} \cdot \mathbf{A}_{1:8}{}^T \cdot (-\mathbf{A}_9)$

# Homography from N ≥ 4 points

**Consider N point correspondences** $p_i = (x_i, y_i) \;\rightarrow\; p'_i = (x'_i, y'_i)$

$$\mathbf{p}'_i = \mathbf{H}\mathbf{p}_i \quad \Rightarrow \quad \boxed{\underset{2N\text{x}9}{\mathbf{A}} \cdot \underset{9\text{x}1}{\mathbf{h}} = \underset{2N\text{x}1}{\mathbf{0}}} \qquad (*)$$

for $i = 1,\ldots,N$

over-constrained system

***Approach 2***: **add constraint ||h||=1**

solve $\boxed{\min_{h:||h||=1} ||\mathbf{A} \cdot \mathbf{h}||}$ (*homogeneous least-squares*)

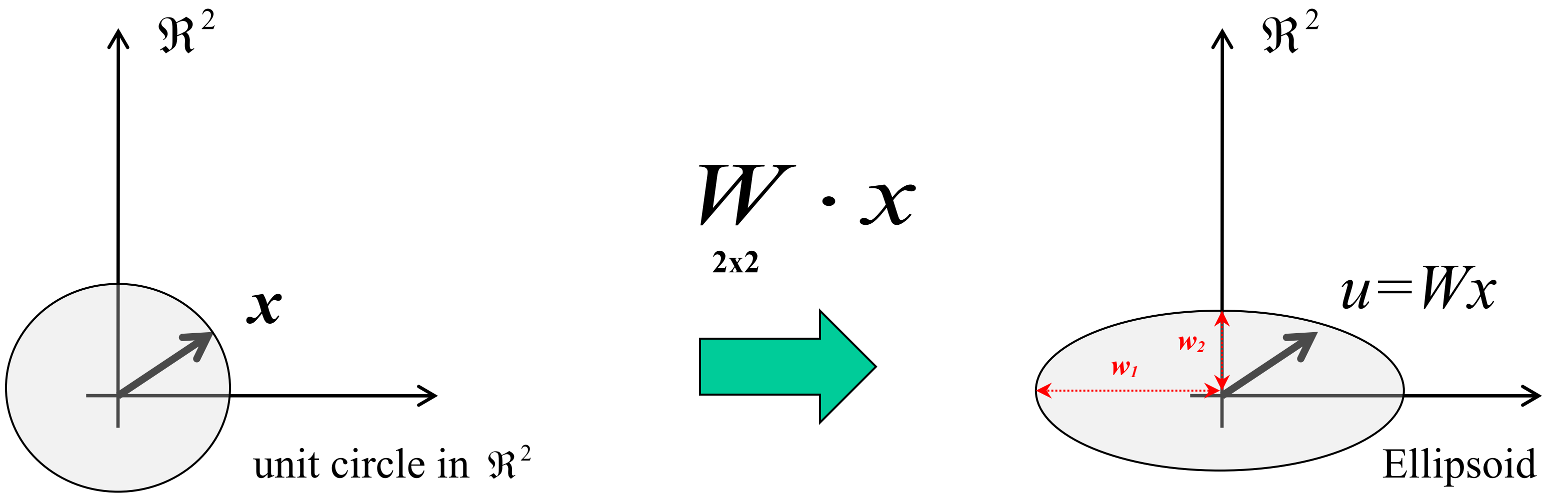
**Solution**: (unit) eigenvector of $\mathbf{A}^T\mathbf{A}$ corresponding to the smallest eigen-value (use SVD, see next slide)

DLT method (see p.91 in Hartley and Zisserman)

# Simple motivating example:

Consider 2x2 diagonal matrix $W = \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix}$

**solve:** $\boxed{\min_{x:\|x\|=1} \| W \cdot x \|}$




$\|x\| = 1$ or $x^T \cdot x = 1$

$$x \cdot x^T = 1 \Leftrightarrow u^T W^{-2} u = 1$$

$$\frac{u_1^2}{w_1^2} + \frac{u_2^2}{w_2^2} = 1$$

**Solution**: $x = (1,0)$ if $w_1 < w_2$
$x = (0,1)$ if $w_2 < w_1$

$\Leftarrow$ equivalently, solve $\min_{u \in Ellips} \| u \|$

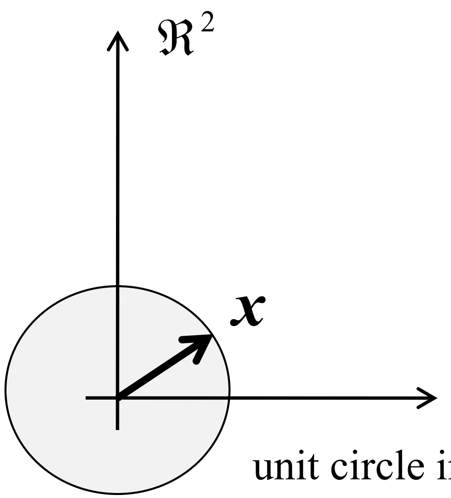
# General case: use SVD (rough idea)

$$\overset{\textit{embed}}{\phantom{x}}\quad\overset{\textit{scale}}{\phantom{x}}\quad\overset{\textit{rotate}}{\phantom{x}}$$

$$A = U \cdot W \cdot V^T$$

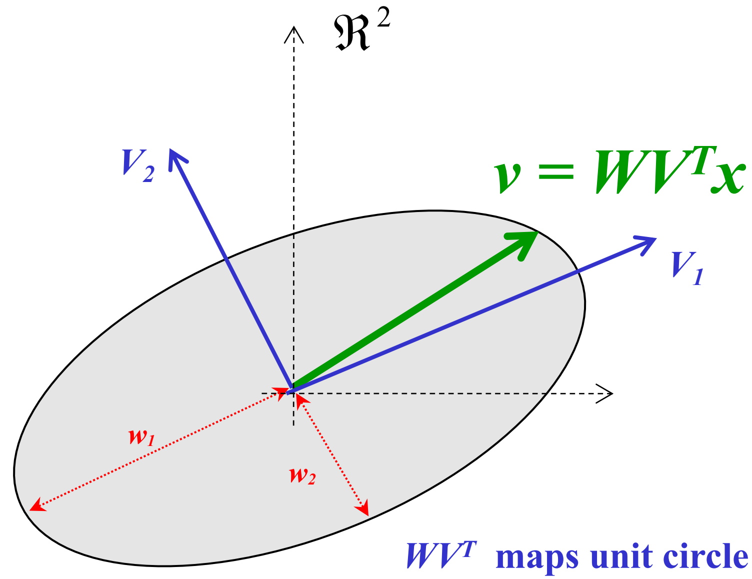
M ≥ N: MxN MxN NxN NxN

where $U$ and $V$ are matrices with ortho-normal columns and $W$ is diagonal with elements $w_i \geq 0$ (see "Numerical Recipes in C", edition 2, Sec. 2.6)




unit circle in $\Re^2$
$||x|| = 1$ or $x^T \cdot x = 1$

$$\underset{2x2}{W}\,\underset{2x2}{V^T} \cdot x$$




**$WV^T$ maps unit circle $x^T \cdot x = 1$**
**onto ellipsoid $v^T W^{-2} v = 1$**
**in basis $\{V_1, V_2\}$**

**check that $x = V_i$ is mapped to point**
**$W V^T x = w_i \, e_i$**

vector with zeros
and 1 in the $i$-th position

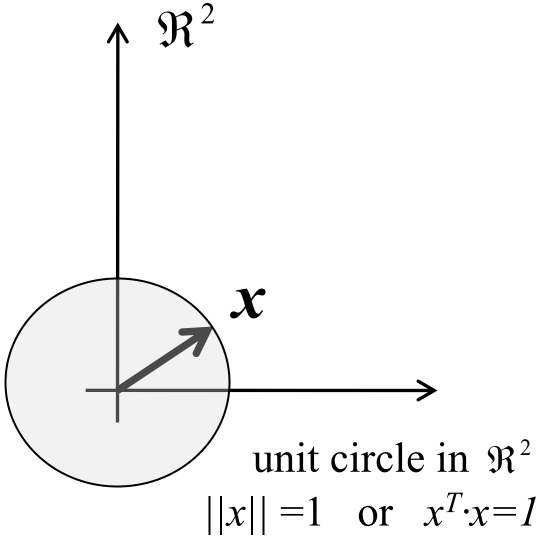
- **can interpret multiplication by $V^T$ as change of ortho-normal basis**
  (or as space rotation, but illustration should be modified)
- **interpret multiplication by $W$ as anisotropic scaling/deformation**

# General case: use SVD (rough idea)

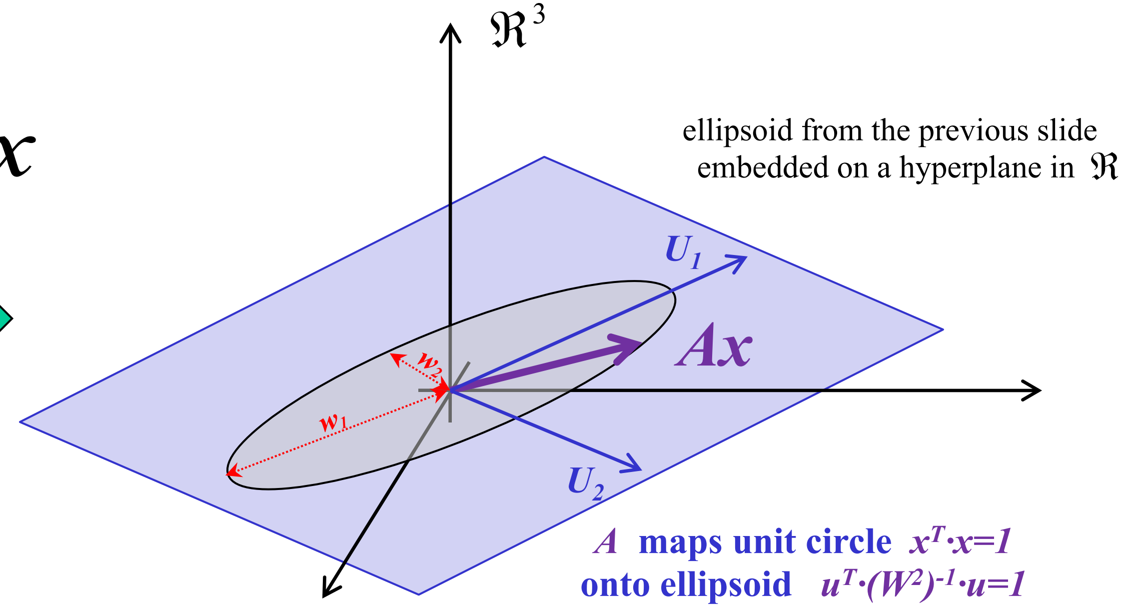
$$\overset{\textit{embed}}{} \quad A = \underset{\text{MxN}}{\overset{\textit{embed}}{U}} \cdot \underset{\text{NxN}}{\overset{\textit{scale}}{W}} \cdot \underset{\text{NxN}}{\overset{\textit{rotate}}{V^T}}$$

M ≥ N: $A$ is MxN, $U$ is MxN, $W$ is NxN, $V$ is NxN

where $U$ and $V$ are matrices with ortho-normal columns and $W$ is diagonal with elements $w_i \geq 0$ (see "Numerical Recipes in C", edition 2, Sec. 2.6)




$$\underset{3\text{x}2}{A} \cdot x$$




**$A$ maps unit circle $x^T \cdot x = 1$**
**onto ellipsoid $u^T \cdot (W^2)^{-1} \cdot u = 1$**
**in the range of $A$ (2D subspace of $\Re^3$)**

**Indeed, the coordinates of vector $u$ are in basis $\{U_1, U_2\}$, i.e. $u = WV^T \cdot x$ and $x = VW^{-1} \cdot u$**

**How does SVD help to solve** $\min\limits_{x:||x||=1} ||\mathbf{A} \cdot x||$ **?**

$$A \cdot V_i = UWV^T \cdot V_i = UW \cdot e_i = w_i \cdot (U \cdot e_i) = w_i \cdot U_i \qquad \Rightarrow ||A \cdot V_i|| = w_i$$

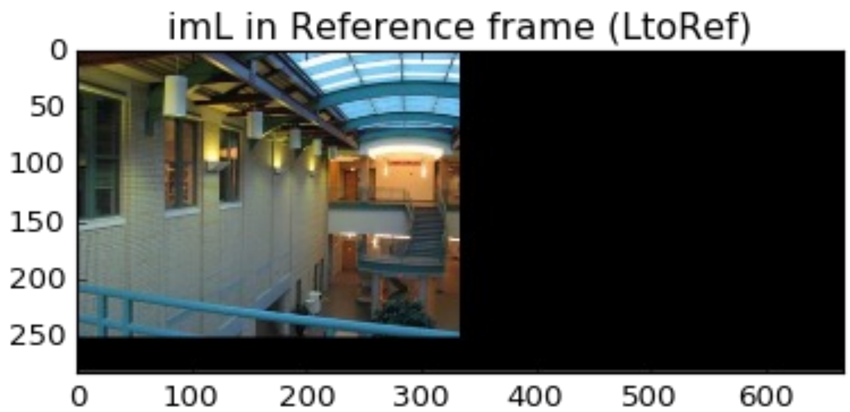
**vector $x = V_i$ corresponding to the least $w_i$ solves the problem**

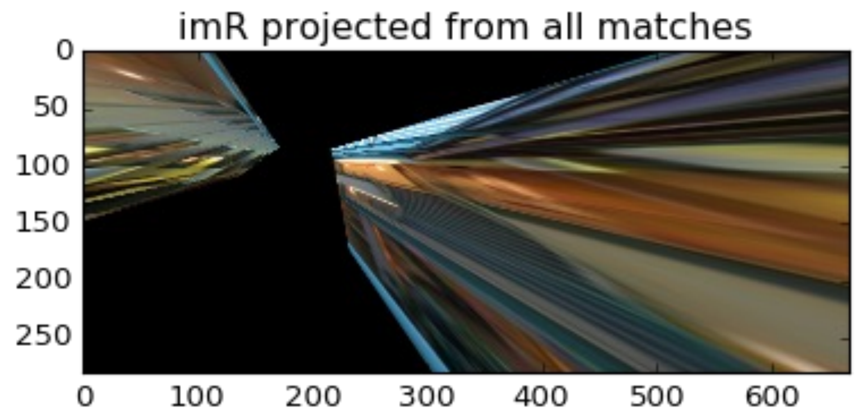
Check that $V_i$ and $(w_i)^2$ are eigen vectors/values for matrix $A^T A$ (note: ellipsoid $x^T \cdot (A^T A) \cdot x = 1$ maps onto circle $u^T \cdot u = 1$)

$$A^T A \cdot V_i = VW^2 V^T \cdot V_i = VW^2 \cdot e_i = w_i^2 \cdot V_i$$

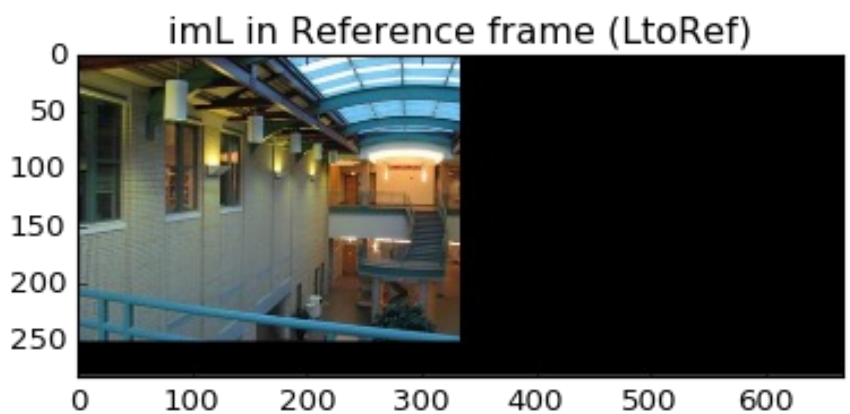
=> **can use eigen decomposition of $A^T A$ instead of SVD of $A$.**

# Least squares fail in presence of outliers

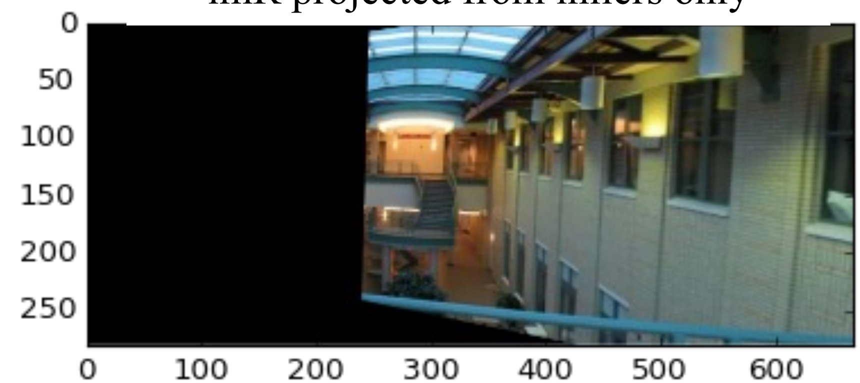


# Least squares work if using “inliers” only ( detecting these? – soon )




imR projected from inliers only



# Least squares work if using "inliers" only ( detecting these? – soon )


larger errors
in the area
with no matches

**"algebraic errors"**

**we minimized**

$$ax + by + c - gxx' - hyx' - ix' \approx 0$$
$$dx + ey + f - gxy' - hyy' - iy' \approx 0$$

**"geometric errors"**

**observable in the image**

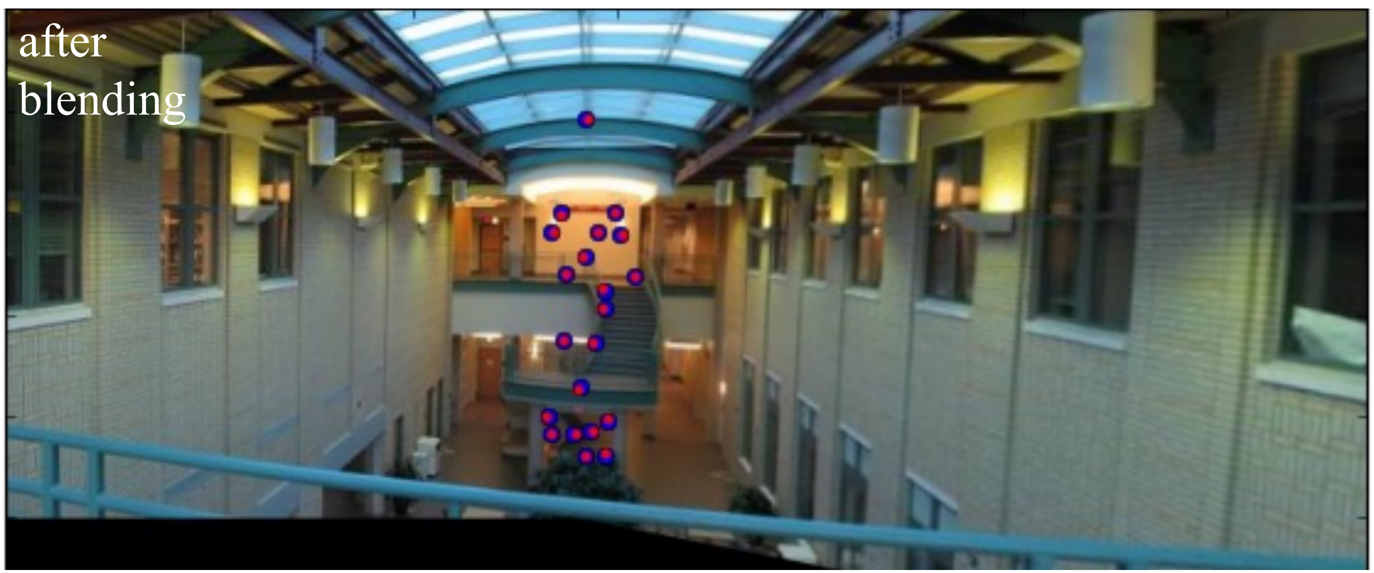
$$x' - \frac{ax + by + c}{gx + hy + i} \approx 0$$
$$y' - \frac{dx + ey + f}{gx + hy + i} \approx 0$$

(harder to minimize)

*"errors"* among inliers $\|p' - Hp\|$

**Did we actually minimize these errors?**

# Least squares work if using "inliers" only ( detecting these? – soon )




**Question: how to remove outliers automatically?**

# RANSAC for robust homography fitting

**only two differences:** **1. need to randomly sample four pairs** $(p, p')$

the minimum number of matches to estimate a homography H

**2. pair** $(p, p')$ **counts as an inlier for a given homography H if**

$$\| p' - Hp \| \leq T$$

# RANSAC for robust homography fitting

**Homography for corrupted four matches is likely to have only a few inliers** $(p, p')$

**(randomly sampled)**

$$\| p' - Hp \| \leq T$$

# RANSAC for robust homography fitting

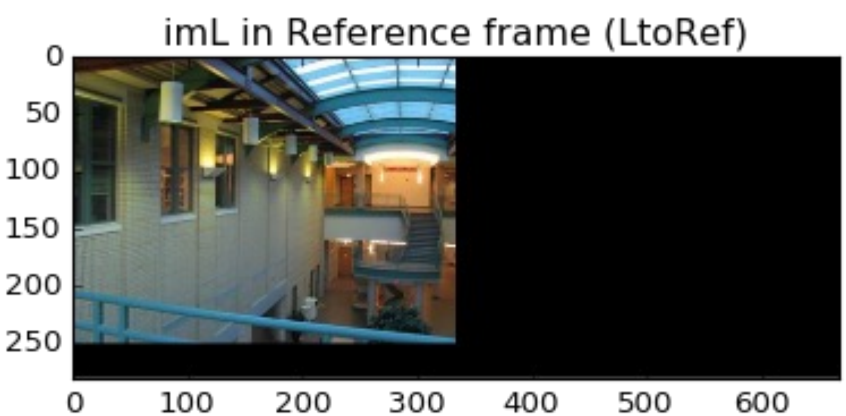
**Homography for good <u>four</u> matches has 21 inliers** $(p, p')$

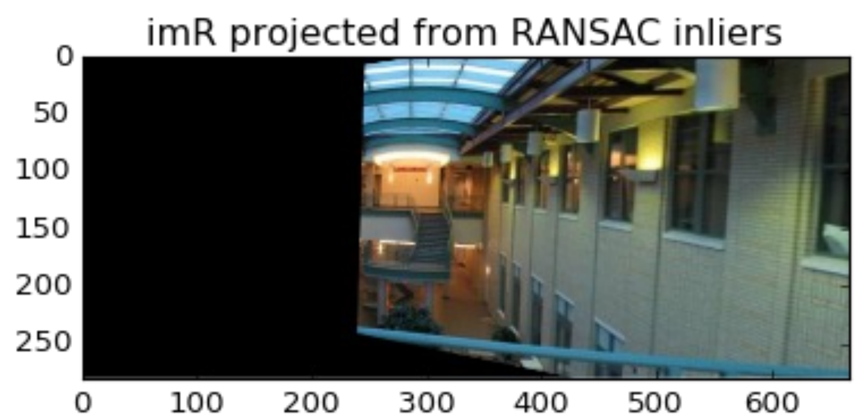
$$\| p' - Hp \| \leq T$$

**(randomly sampled)**

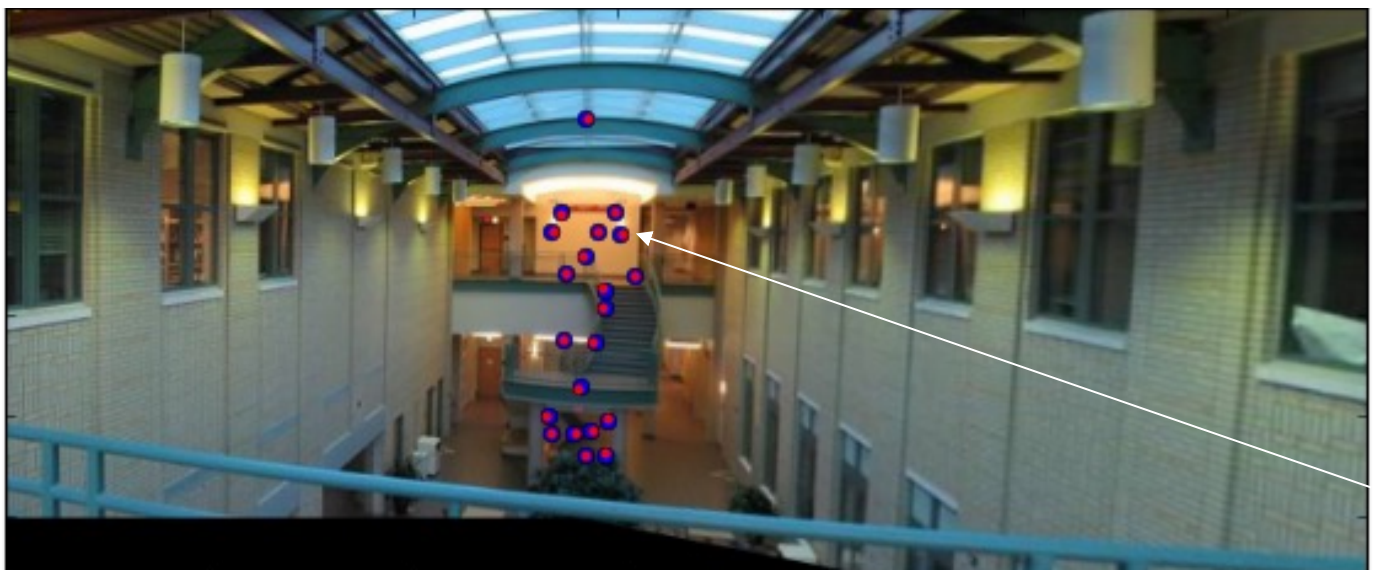
# RANSAC for robust homography fitting

**Inliers for the randomly sampled homography with the largest inlier set**



# RANSAC for robust homography fitting



matched
inliers
$|| \color{blue}{p'} - \color{red}{Hp} ||$

**The final automatic panorama result**

# RANSAC for robust model fitting

**In general (for other models):**
**always sample the smallest number**
**of points/matches needed to estimate a model**

RANSAC loop:

1. Select <u>four</u> feature pairs (at random)
2. Compute homography H (exact)
3. Count *inliers* $(p, p')$ where $\| p' - Hp \| \leq T$

   **e.g. geometric errors**
4. Iterate N times (steps 1-3). Keep the largest set of inliers.
5. Re-compute <u>least-squares</u> H estimate on all of the inliers

   **e.g. for algebraic errors**
   **(for simplicity)**

# Other examples of geometric model fitting

images from **different** view points (optical centers)

**Merton College III data**

**from Oxford's Visual Geometry Group**

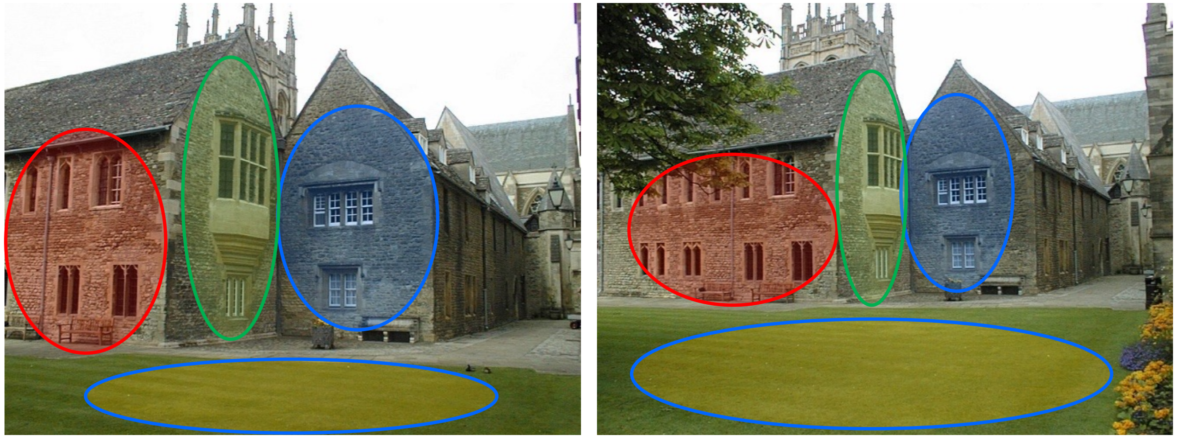
http://www.robots.ox.ac.uk/~vgg/data/data-mview.html

**Question:** Is it possible to create a panorama from these images?
(or, is there a *homography* that can match overlap in these images?)

Can a *homography* map/warp **a part** of the left image onto **a part** of the right image?

# Other examples of geometric model fitting

images from **different** view points (optical centers)



**Merton College III data**

**from Oxford's Visual Geometry Group**

http://www.robots.ox.ac.uk/~vgg/data/data-mview.html

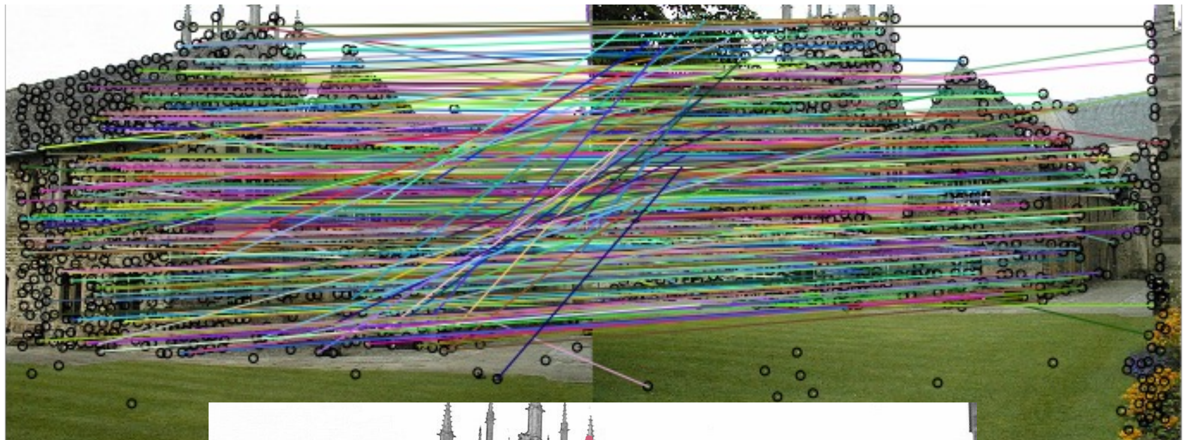
There should be a *homography* for each plane in the scene (Why?)

**Question:** How can we detect such *homographies*?

**What do these multiple *homographies* give us?**

# Other examples of geometric model fitting

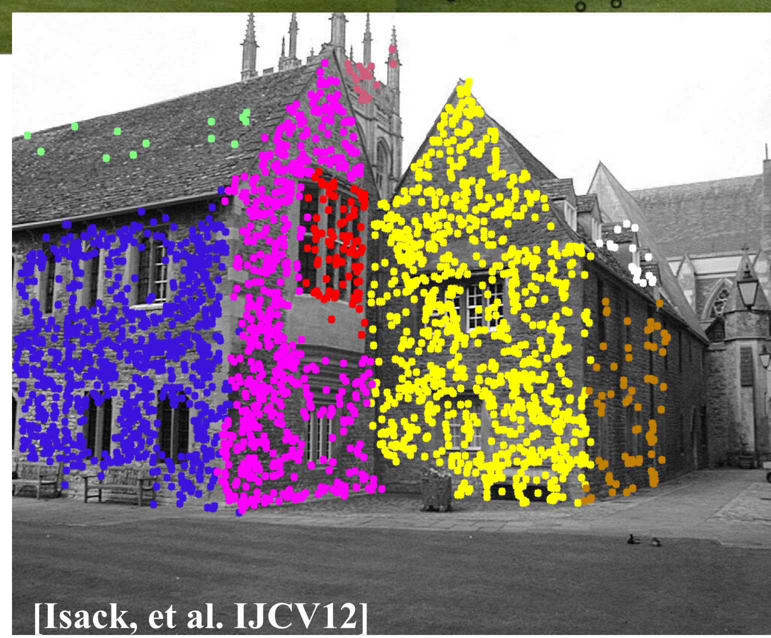
matched features (p,p’), as earlier



NOTE:
**good matches** can be used for reconstructing 3D points if camera positions & orientations are known:

$$(\boldsymbol{p},\boldsymbol{p}') \rightarrow \boldsymbol{X_p} \in \boldsymbol{R^3}$$

(Triangulation, see next topic)




1. Allow to remove bad matches (outliers)

2. Correspondences can be estimated with subpixel precision $(\boldsymbol{p},\boldsymbol{p}') \rightarrow (\boldsymbol{p},\boldsymbol{Hp})$

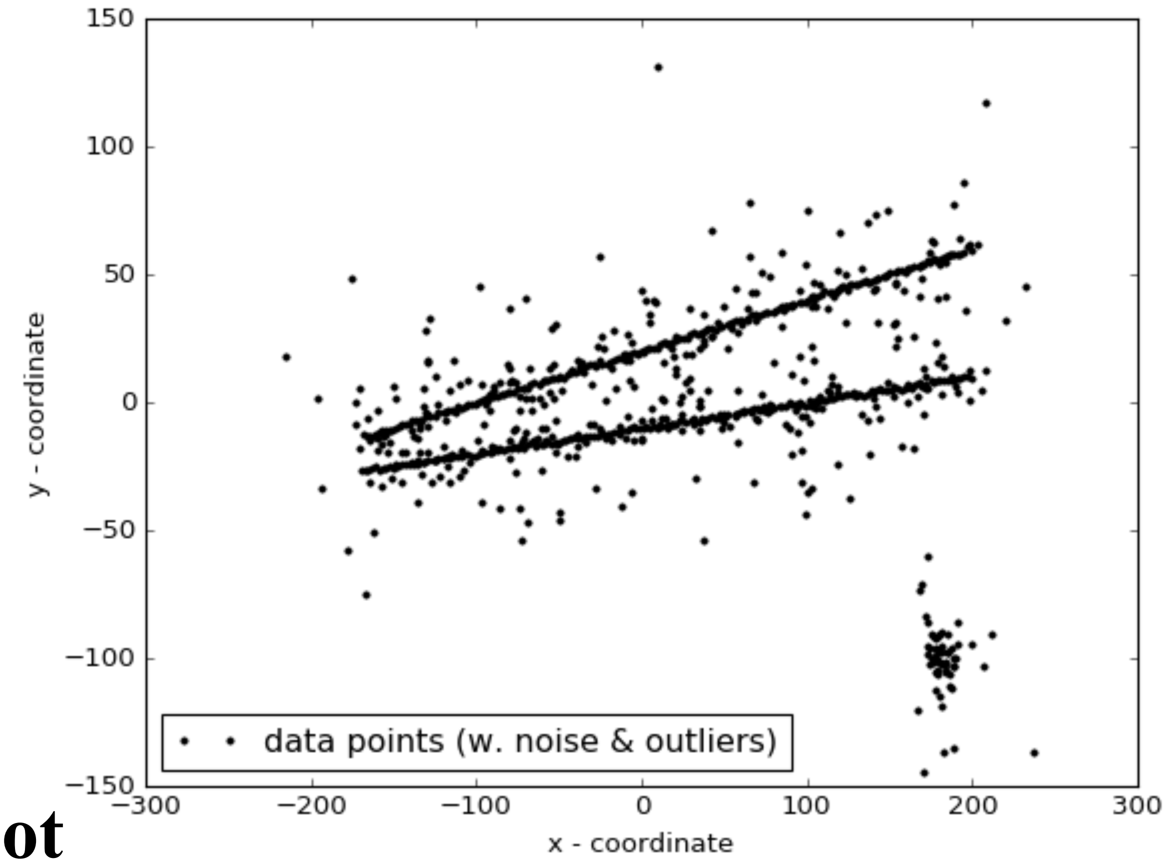
3. Inliers allow to segment planar regions

4. Segmentation allows to extend correspondence from inliers to any point $\boldsymbol{p}$ inside segment: $(\boldsymbol{p},\boldsymbol{Hp})$

5. Piece-wise planar 3D scene reconstruction

**What do these multiple *homographies* give us?**

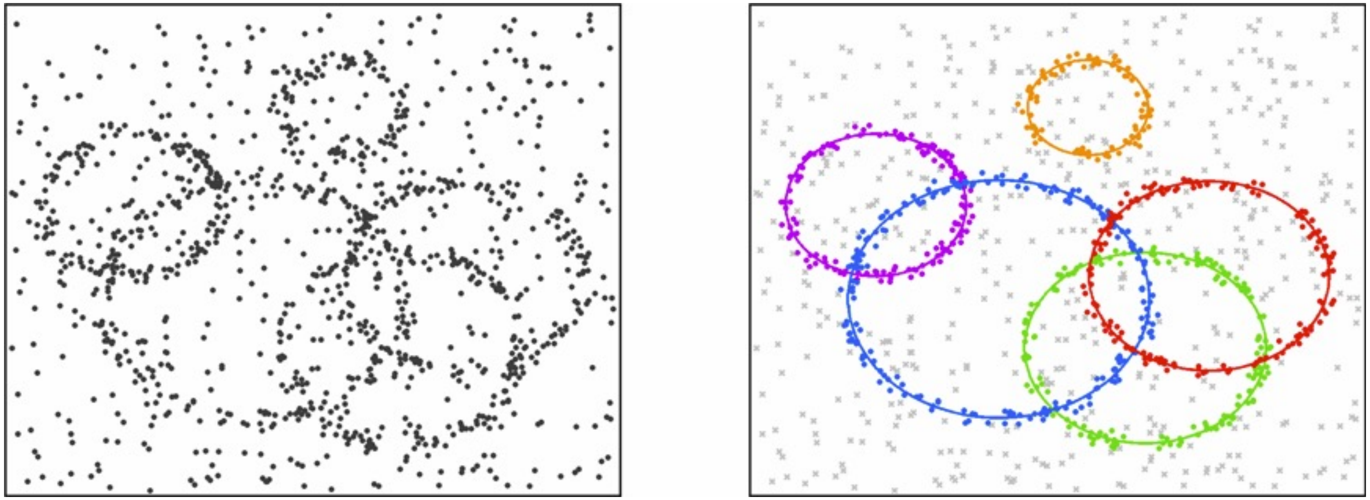
# So, how do we find multiple models?




**Why not**
**RANSAC**
**again?**

# Fitting other geometric models

$$\| p - \theta \| \;:=\; \left( (x_p - c_x)^2 - (y_p - c_y)^2 - r^2 \right)^2 \qquad \text{for} \qquad \theta = \{c_c, c_y, r\}$$



**Model fitting for arbitrary geometric models $\boldsymbol{\theta}$**

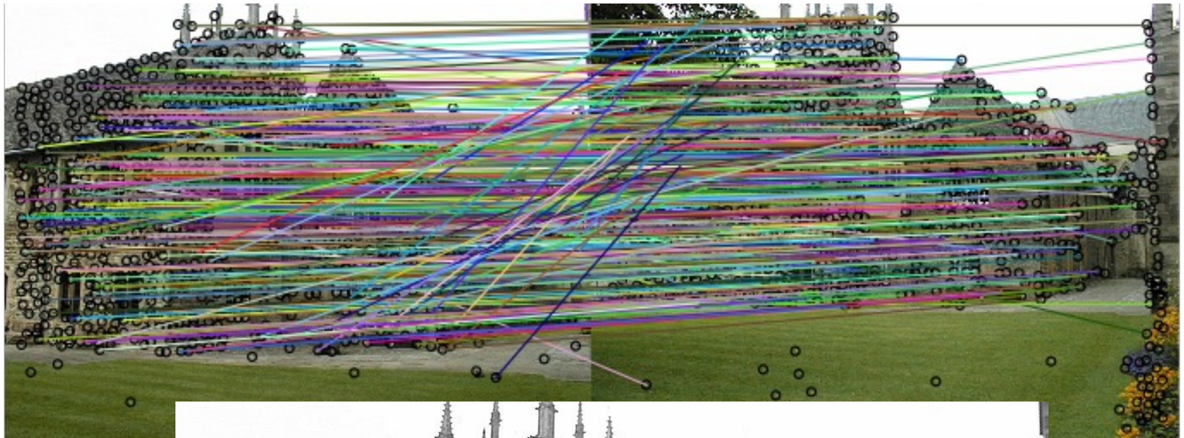
**Need:** 1) define an error measure w.r.t. model parameters $\| p - \theta \|$

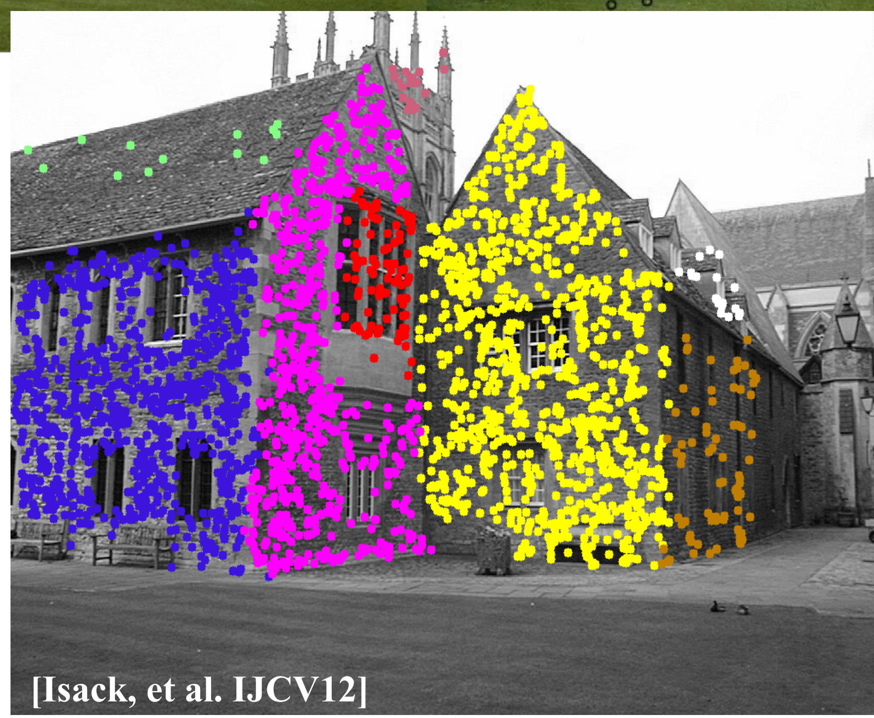
2) efficient method for minimizing the sum of errors among inliers w.r.t. model parameters $\boldsymbol{\theta}$

$$\min_{\theta} \sum_{p \in S_\theta} \| p - \theta \|$$

# Fitting multiple homographies (e.g. planes)

matched features (p,p'), as earlier



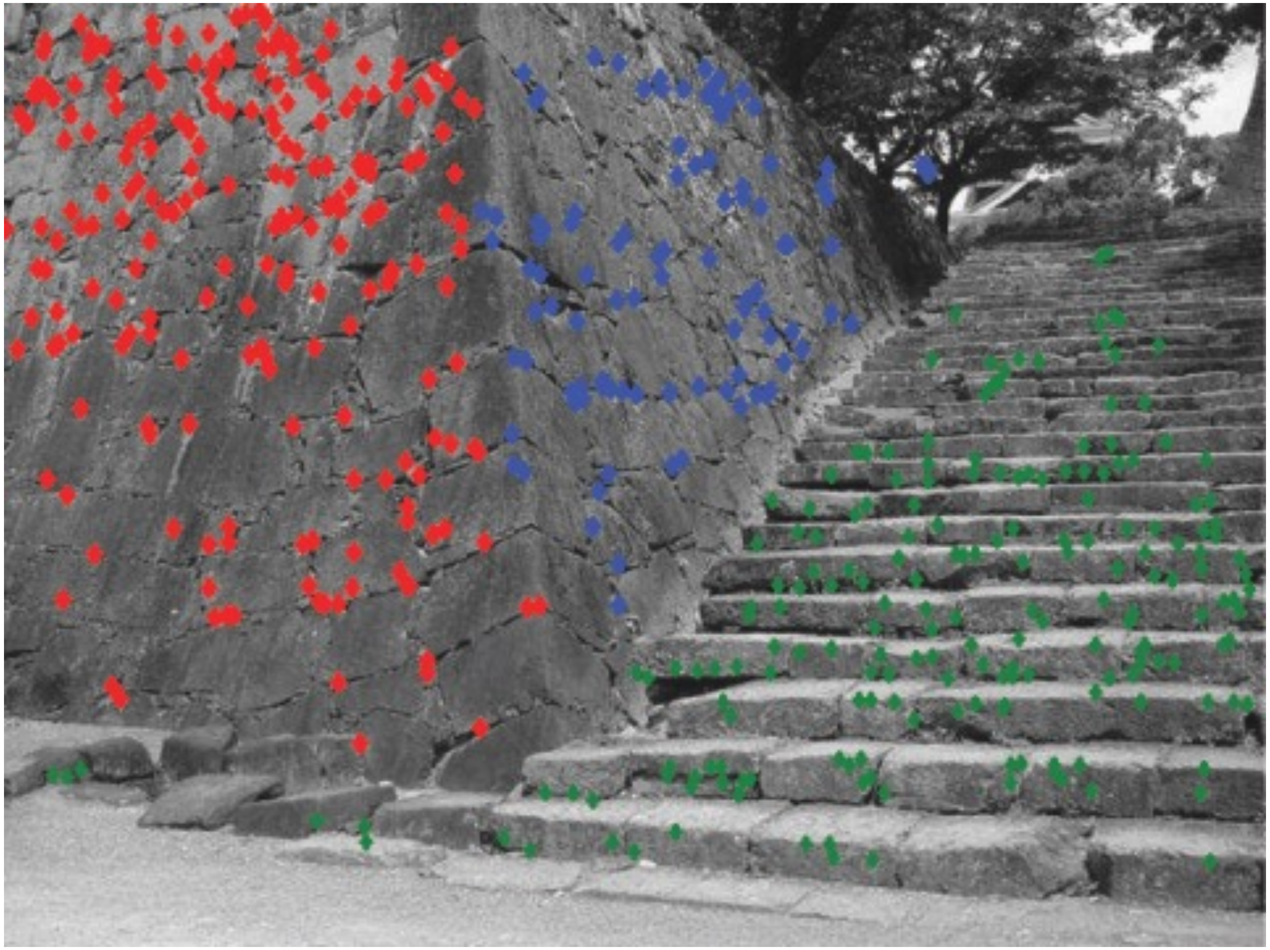



using **symmetric re-projection errors**

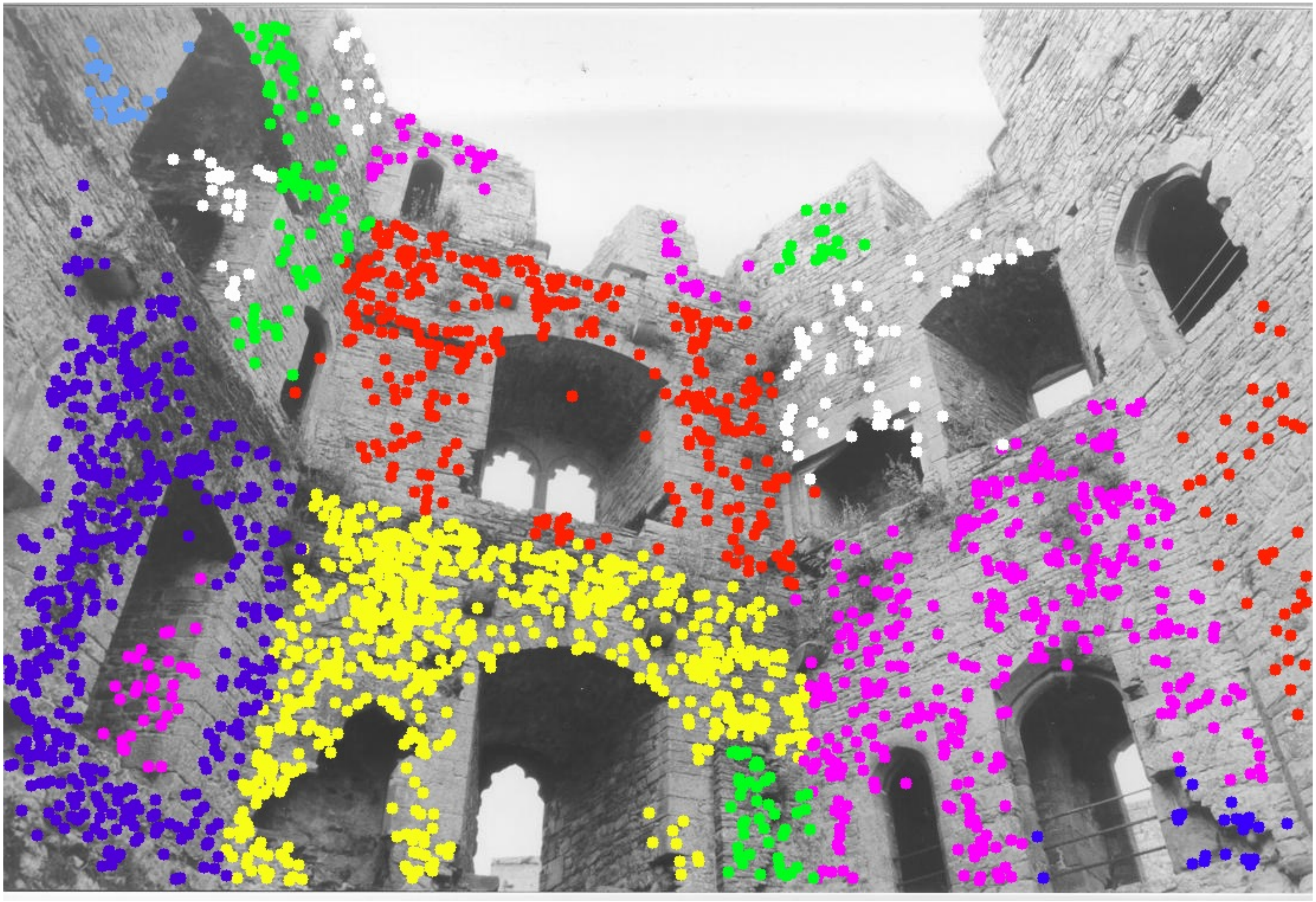
$$\| p' - Hp \| + \| p - H^{-1} p' \|$$

as an error measure between match ($p$, $p'$) and homography $H$

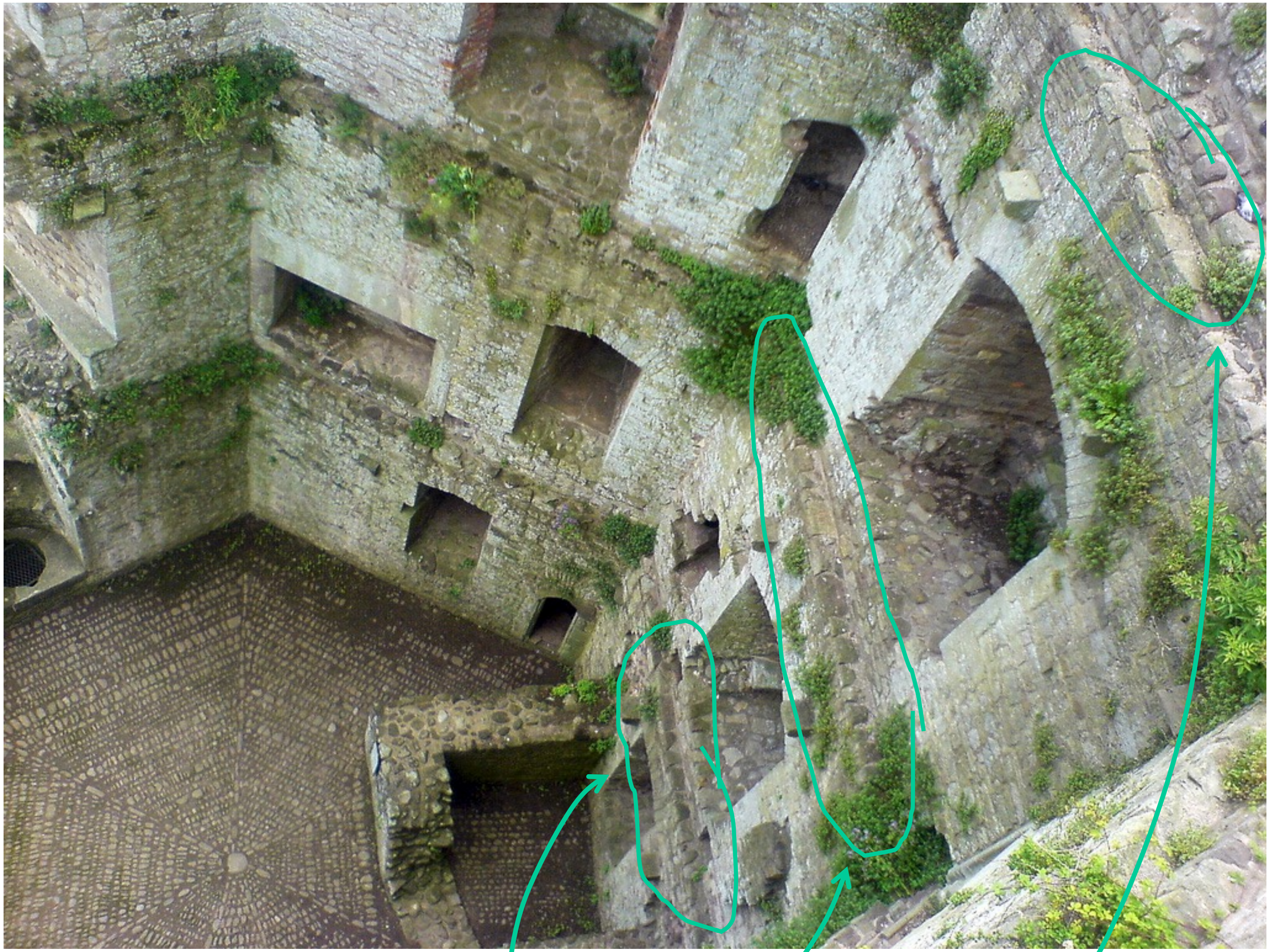
# Fitting multiple homographies (e.g. planes)



# Fitting multiple homographies (e.g. planes)



# same scene from a different view point…



**Note very small steps between each floor**

# Geometric model fitting in vision

**MODELS: lines, planes, homographies, affine transformations, projection matrices, fundamental/essential matrices, etc.**

**next topic**

- **single models** (e.g. panorama stitching, camera projection matrix)
- **multiple models** (e.g. multi-plane reconstruction, multiple rigid motion)

**FIRST STEP: detect some features (corners, LOGS, etc) and compute their descriptors (SIFT, MOPS, etc.)**

**SECOND STEP: match** or **track**

**THIRD STEP: fit models**
(minimization or errors/losses)